

ESE 2027

Main Examination

UPSC ENGINEERING SERVICES EXAMINATION

Topicwise

Conventional Practice Book

Civil Engineering

PAPER-II





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**ESE Main Examination • Conventional Practice Book :
Civil Engineering PAPER-II**

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01

Fluid Mechanics, Open Channel Flow & Hydraulic Machines

1. Fluid Properties

Level-1

- 1.1** A 6 cm diameter soap bubble is to be enlarged by blowing air into it. Taking the surface tension of soap solution to be 0.039 N/m. Determine the work input required to inflate the bubble diameter to 6.9 cm.

[5 Marks]

Sol:

$$\text{Initial surface area} = 4\pi r_1^2$$

$$\text{Final surface area} = 4\pi r_2^2$$

$$\text{Change in surface area } (\Delta A) = 4\pi(r_2^2 - r_1^2)$$

$$\begin{aligned} \text{Work required} &= \text{Change in surface energy} \\ &= T \cdot \Delta A \end{aligned}$$

$$W = \frac{0.039 \times 4\pi}{10^4} \left[\left(\frac{6.9}{2} \right)^2 - \left(\frac{6}{2} \right)^2 \right] = 1.42 \times 10^{-4} \text{ N-m}$$

- 1.2** A Glass tube of 2 mm internal diameter is immersed in an oil of mass density 960 kg/m³ to a depth of 10 mm. If a pressure of 172 N/m² is needed to form a bubble which is just released. Determine the surface tension of oil.

[10 Marks]

Sol:

$$\text{External pressure on bubble, } \rho gh = 960 \times \frac{9.81 \times 10}{1000} = 94.176 \text{ N/m}^2$$

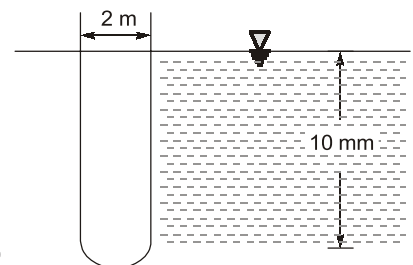
$$\text{Inside pressure of bubble} = 172 \text{ N/m}^2$$

$$\text{Difference in pressure, } \Delta P = 172 - 94.176 = 77.824 \text{ N/m}^2$$

$$\Delta P \text{ of bubble} = \frac{2T}{R} \quad (\text{Only one contact layer})$$

$$77.824 = \frac{2 \times T}{1 \times 10^{-3}}$$

$$\text{Solving,} \quad T = 0.0389 \text{ N/m}$$



- 1.3** As the nutrients are carried to upper parts of a plant by tiny tubes due to action of capillarity. Determine how high will the water solution will rise in a tree of 0.002 mm diameter tube as a result of the capillary effect.

Assume the water solution at 20°C

Contact angle = 15°

[5 Marks]

Sol:

The rise in a capillary is given by

$$h = \frac{2T \cos \theta}{\rho r g}$$

where T = Surface tension = 0.073 N/m (at 20°C)

θ = Contact angle = 15°C (given)

$$r = \text{Radius of capillary tube} = \frac{0.002}{2} = 0.001 \text{ mm}$$

Putting,

$$h = \frac{2 \times 0.073 \times \cos 15^\circ}{10^3 \times 0.001 \times 10^{-3} \times 9.81}$$

$$h = 14.38 \text{ m} \quad (\text{Ans})$$

- 1.4** A spherical shell made of a material with a density of 1600 kg/m³ is placed in water. If the inner and outer radii of shell are $R_1 = 6 \text{ cm}$, $R_2 = 8 \text{ cm}$ determine the percentage of the shell's total volume that would be submerged.

[10 Marks]

Sol:

$$\text{Volume of shell} = \frac{4}{3} \pi (R_2^3 - R_1^3)$$

$$\text{Given } R_1 = 6 \text{ cm}, R_2 = 8 \text{ cm}$$

$$\text{Total weight of shell} = 1800 \times g \times \frac{4}{3} \pi (0.08^3 - 0.06^3)$$

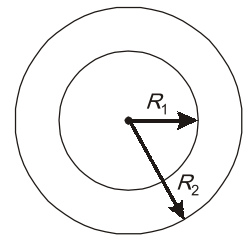
$$\begin{aligned} \text{Also, the Buoyant force} &= \text{Weight of displaced fluid} \\ &= \text{Volume displaced} \times g \times 1000 \end{aligned}$$

Now, equating,

$$1600 \times g \times \frac{4}{3} \pi (0.08^3 - 0.06^3) = \text{Volume displaced} \times g \times 1000$$

$$\text{Volume displaced} = 1982.80 \times 10^{-6} \text{ m}^3 = 1982.8 \text{ cm}^3$$

$$\% \text{ submerged} = \frac{1982.8 \text{ cm}^3}{\frac{4}{3} \pi \times (8)^3} \times 10 = 92.5\%$$



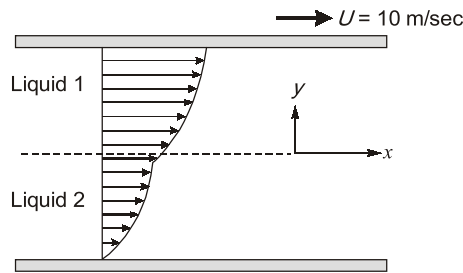
Level-2

- 1.5** Two immiscible Newtonian liquids flow steadily between two large parallel plates under the influence of an applied pressure gradient. The lower plate is fixed while the upper plate is pulled with a constant velocity of $U = 10 \text{ m/sec}$. Thickness, h of each layer on fluid is 0.5 m. The velocity profile of each layer is given by

$$\begin{aligned} V_1 &= 6 + ay - 3y^2 & -0.5 \leq y \leq 0 \\ V_2 &= b + cy - 9y^2 & 0 \leq y \leq +0.5 \end{aligned}$$

Where a , b and c are constant

- Determine the values of constants a , b and c .
- Develop an expression for viscosity ratio $(\mu_1/\mu_2) = ?$
- Determine the forces and their directions exerted by liquids on both plates if $\mu_1 = 10^{-3} \text{ Pa sec}$ and each plate has a surface area of 4 m².



[15 Marks]

Sol:**(a)**For $-0.5 \leq y \leq 0$

$$V = 6 + ay - 3y^2$$

at $y = -0.5 \Rightarrow y = 0$

$$6 + a(-0.5) - 3(-0.5)^2 = 0$$

Solving

$$a = 10.5$$

For $0 \leq y \leq +0.5$ $\Rightarrow$

$$V = b + cy - 9y^2$$

at $y = 0.5$ m, $v = 10$ m/sec (No slip condition)

$$y = 0.5 \text{ m}$$

$$b + c(0.5) - 9(0.5)^2 = 10$$

$$b + 0.5c = 12.25$$

Also at interface,

$$V_1 = V_2$$

$$6 + 10.5(0) - 3(0) = b + c(0) - 9(0)^2$$

$$b = 6$$

$$c = 12.5$$

(b)

$$V_1 = 6 + 10.5y - 3y^2$$

$$V_2 = 6 + 12.5y - 9y^2$$

Now at interface shear stress is equal

$$\tau_1 = \tau_2$$

$$\mu_2 \frac{\partial V_1}{\partial y} = \mu_1 \frac{\partial V_2}{\partial y}$$

$$\frac{\mu_1}{\mu_2} = \left[\frac{10.5 - 6y}{12.5 - 18y} \right]_{at y=0} = \frac{10.5}{12.5} = 0.84$$

(c)Force on bottom plate $= \tau_0 \times \text{Area}$

$$\tau_0 = \mu_2 \left| \frac{\partial V_1}{\partial y} \right|_{at y=-0.5 \text{ m}} = \frac{10^{-3}}{0.84} [10.5 - 6y]_{y=-0.5 \text{ m}} = 16.07 \times 10^{-3} \text{ N/m}^2$$

$$\text{Force} = 16.07 \times 10^{-3} \times 4 = 64.28 \times 10^{-3} \text{ N}$$

On top plate,

$$\begin{aligned} \tau_0 &= \mu_1 \left| \frac{\partial V_2}{\partial y} \right|_{at y=+0.5 \text{ m}} \\ &= 10^{-3} [12.5 - 18y]_{y=0.5 \text{ m}} \\ &= 10^{-3} [12.5 - 18 \times 0.5] = 3.5 \times 10^{-3} \text{ N/m}^2 \end{aligned}$$

$$\text{Force} = 14 \times 10^{-3} \text{ N} \quad \text{(Ans)}$$

2. Fluid Pressure Measurements, Buoyancy and Rigid Body Motion

Level-1

- 2.1** A 10g of wood of square section $0.36 \text{ m} \times 0.36 \text{ m}$ and specific gravity 0.8 floats in water one edge is depressed and released causing the dog to roll. Estimate the period of the roll.

[10 Marks]

Sol:

Since time period is given by,

$$T = 2\pi \sqrt{\frac{K^2}{g \times GM}}$$

where, $GM = \text{Metacentric height} = \frac{I}{V_{Dis}} - B_G$

Considering 1 m length of log

For vertical equilibrium

$$0.8 \times \gamma_w \times 0.36 \times 0.36 \times 1 = \gamma_w \times 0.36 (x) \times 1$$

Solving, $x = 0.288 \text{ m}$

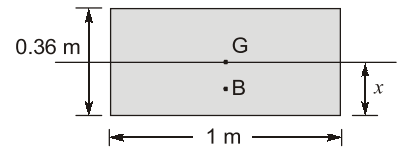
$$BG = \frac{0.36}{2} - \frac{0.288}{2} = 0.036 \text{ m}$$

$$I = \frac{1 \times 0.36^3}{12} \quad V_{Dis} = 0.36 \times 1 \times 0.288$$

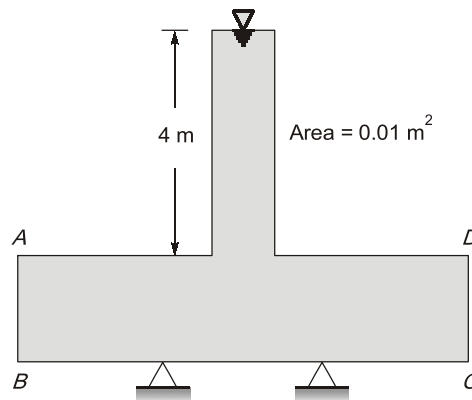
$$\text{Metacentric height, } GM = \frac{1 \times 0.36^3}{12 \times 0.36 \times 1 \times 0.288} - 0.036 = 1.5 \times 10^{-3}$$

$$k = \text{Radius of gyration} = \sqrt{\frac{I}{A}} = \sqrt{\frac{1 \times 0.36^3}{12 \times 1 \times 0.36}} = 0.104 \text{ m}$$

$$\text{Putting, } T = 2\pi \sqrt{\frac{0.104^2}{9.81 \times 1.5 \times 10^{-3}}} = 5.384 \text{ sec}$$



- 2.2** Water rises to level 4 m in the pipe attached to the tank ABCD shown in figure.



- (i) Compute the total pressure on the bottom of the tank having area $6.5 \text{ m} \times 2.5 \text{ m}$.
- (ii) Compare the total weight of water with the result in (i).

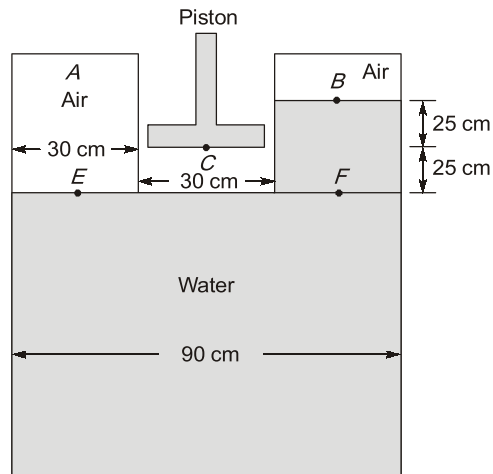
[8 Marks]

Sol:

- (i) Pressure intensity on bottom BC is uniform and is equal to
 $= 9.81 \times (4 + 2) = 58.86 \text{ kN/m}^2$
 Total force acting $= 58.86 \times (6.5 \times 2.5)$
 $= 956.475 \text{ kN}$
- (ii) Total weight of water $= 9.81 (6.5 \times 2.5 \times 2 + 4 \times 0.01)$
 $= 319.217 \text{ kN}$

Thus the total weight of water is much less than force due to total pressure, this is known as Pascal's paradox.

- 2.3** Two chambers with the same fluid at their base are separated by a 30 cm diameter piston whose weight is 25 N, as shown in figure. Calculate the gauge pressure in chambers A and B.



[10 Marks]

Sol:

For left limb /Arm :

Equating pressure at the level of E

$$P_A = \frac{25 \text{ N}}{\frac{\pi}{4} \times \left(\frac{30}{100}\right)^2} + \gamma_w \times 0.25$$

Solving, $P_A = 353.86 \text{ N/m}^2 + 9810 \times 0.25$

$$P_A = 2806.36 \text{ N/m}^2$$

For right arm

Equating pressure at level of C

$$P_B + \gamma_w \times 0.25 = \frac{25}{\frac{\pi}{4} \times (0.3)^2}$$

Solving,

$$P_B = 353.86 \text{ N/m}^2 - 9810 \times 0.25 = -2098.64 \text{ N/m}^2 \quad (\text{Vacuum})$$

- 2.4** An inclined manometer gives a reading of 5 cm along the tube inclined at 15° to the horizontal. The measuring liquid has a specific gravity of 1.45. Find the pressure applied on the basin side taking into account the correction applied due to the change in level of the liquid in the basin which has a diameter of 5 cm and that of inclined tube 5 mm.

[6 Marks]

Sol:

$$\text{Effective vertical reading} = 5 \sin 15^\circ + \Delta h$$

By conservation of volume

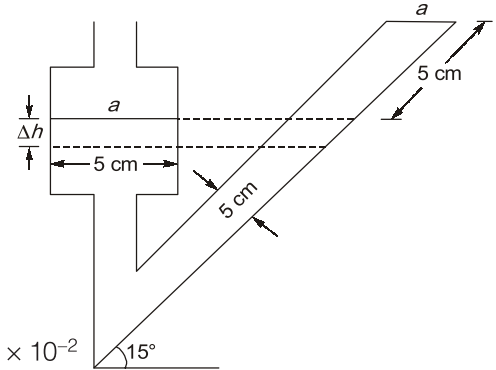
$$\frac{\pi}{4} \times \Delta h \times (50)^2 = \frac{\pi}{4} \times 5^2 \times 5$$

Solving $\Delta h = 0.05 \text{ cm}$

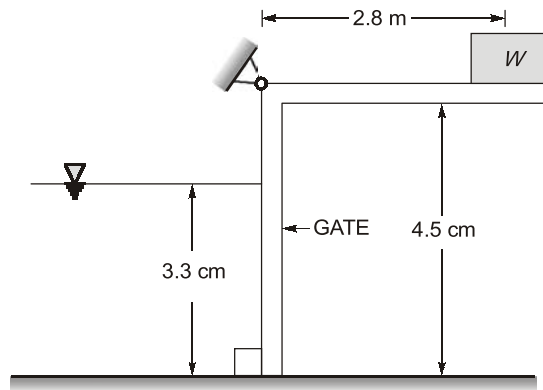
$$\begin{aligned} \text{Effective reading} &= 5 \sin 15^\circ + 0.05 \\ &= 1.344 \text{ cm} \end{aligned}$$

Hence,

$$\begin{aligned} \text{Pressure applied, } P &= \rho_m g h_{\text{eff}} \\ P &= 1.45 \times 10^3 \times 9.81 \times 1.344 \times 10^{-2} \\ &= 191.17 \text{ N/m}^2 \quad (\text{Ans}) \end{aligned}$$



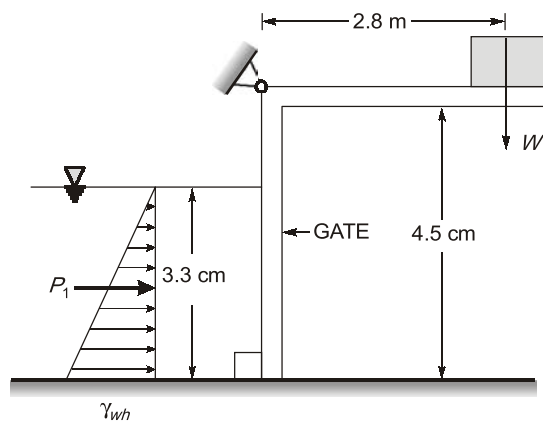
- 2.5** The flow of water from a reservoir is controlled by a 2 m wide L-shape gate hinged at point A, as shown in figure. If it is desired that the gate open when the water height is 3.3 m. Determine the mass of the required weight W .



[10 Marks]

Sol:

Considering hydrostatic distribution of pressure,



$$P_1 = \frac{1}{2} \times \gamma_w \times h \times h \times B = \frac{1}{2} \times 9.81 \times 3.3 \times 3.3 \times 2 = 106.83 \text{ kN}$$

$$\text{Location of } P_1 = \frac{3.3}{3} = 1.1 \text{ m} \quad (\text{from bottom})$$

$$\text{Distance from hinge} = 4.5 - 1.1 = 3.4 \text{ m}$$

Now, equating the moment, about hinge

$\Rightarrow$

$$\Sigma M_{\text{hinge}} = 0$$

$$106.83 \times 3.4 = W \times 2.8$$

Solving,

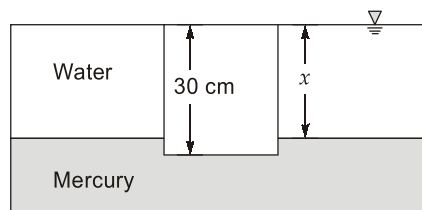
$$W = 129.72 \text{ kN}$$

$$m = \frac{129.72 \times 10^3}{9.81}$$

$$m = 13223.6 \text{ kg}$$

- 2.6** A metallic cube 30 cm side and weighing 500 N is lowered into a tank containing a two fluid layer of water and mercury. Top edge of the cube is at water surface. Determine the position of block at water-mercury interface when it has attained equilibrium.

[12 marks]

Sol:

Let the top edge of the cube be at a distance x from the water-mercury interface.

Thus, the force of buoyancy is given by

$$F_B = \text{Volume of cube in water} \times \rho_w g + \text{Volume of cube in mercury} \times \rho_{Hg} \times g$$

$$F_B = \left(\frac{30}{100} \times \frac{30}{100} \times x \right) \times 1000 \times 9.81 + \frac{30}{100} \times \frac{30}{100} \times \left(\frac{30}{100} - x \right) \times 13600 \times 9.81$$

 $\Rightarrow$

$$F_B = 882.9x + 12007.44 (0.3 - x)$$

Given,

$$\text{Weight of cube, } W = 500 \text{ N}$$

 $\therefore$

$$W = F_B$$

 $\Rightarrow$

$$500 = 882.9x + 12007.44 (0.3 - x)$$

 $\Rightarrow$

$$500 = 882.9x + 3602.232 - 12007.44 x$$

 $\Rightarrow$

$$11124.54x = 3102.232$$

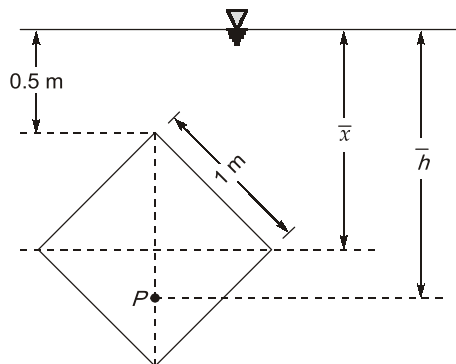
 $\Rightarrow$

$$x = 0.2789 \text{ m}$$

$$= 27.89 \text{ cm}$$

[From the top edge of the block]

- 2.7** A square plate of side 1 m is immersed in water with its diagonal vertical. The upper corner is 0.5 m below the free surface, as shown in figure below. Find the hydrostatic force on the plate and depth of centre of pressure from free surface of water.



[12 marks]

Sol:

Given side of square plate,

$$L = 1 \text{ m}$$

 $\therefore$

$$\text{Area, } A = L^2 = 1 \text{ m}^2$$

$$\bar{x} = \frac{L}{\sqrt{2}} + 0.5 = 1.207 \text{ m}$$

Hydrostatic force on plate,

$$F_x = \rho g A \bar{x}$$

$$= 1000 \times 9.81 \times 1 \times 1.207 = 11840.67 \text{ N}$$

Depth of centre of pressure,

$$\bar{h} = \bar{x} + \frac{I_{CG} \sin^2 \theta}{A \bar{x}}$$

where, $\theta = 90^\circ$

and

$$I_{CG} = \frac{L^4}{12} = \frac{1}{12} = 0.083 \text{ m}^4$$

 $\therefore$

$$\bar{h} = 1.207 + \frac{0.083}{1 \times 1.207} = 1.276 \text{ m from free surface of water}$$

Level-2

- 2.8** An open tank of oil 5 m long contains 2 m of oil (specific gravity 0.8). If the tank accelerated upto 30° include planes at 3.6 m/sec^2 , what is the angle of the oil surface makes with the horizontal? Also find the pressure intensities at the bottom of the vessel at the front and the rear ends? [Assume container horizontal]

[15 Marks]

Sol:

Acceleration in horizontal direction,

$$a_x = 3.6 \cos 30^\circ = 3.12 \text{ m/sec}^2$$

Acceleration in vertical direction,

$$a_y = 3.6 \sin 30^\circ = 1.8 \text{ m/sec}^2$$

Now using,

$$\tan \theta = \frac{a_x}{a_y + g} = \frac{3.12}{1.8 + 9.81} = 0.2687 \text{ m/sec}^2$$

$$\theta = 15^\circ 2' \text{ with the horizontal}$$

Depth at the rear (deep) end is

$$h_1 = 2 + \left(\frac{5}{2} \tan 15^\circ 2' \right) = 2.67 \text{ m}$$

Pressure intensity at the bottom of tank at rear end is

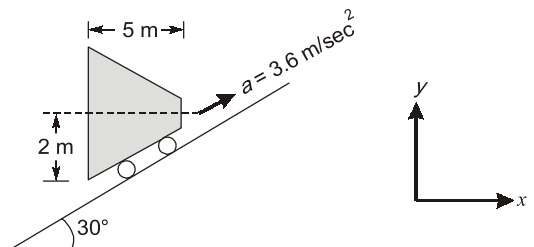
$$P_B = \rho (a_y + g) h_1 = 1000 (1.8 + 9.81) \times 0.8 \times 2.67 = 24.8 \text{ kN/m}^2$$

Similarly the depth of oil in front (shallow) end is

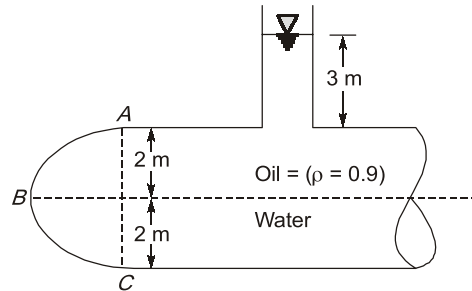
$$h_2 = 2 - \left(\frac{5}{2} \tan 15^\circ 2' \right) = 1.33 \text{ m}$$

Pressure intensity at the bottom of the tank of the front end is

$$P_D = \rho (a_y + g) h_2 = 800 (1.8 + 9.81) \times 1.33 = 12.35 \text{ kN/m}^2$$



- 2.9** The cylindrical tank in figure has a hemispherical end cap ABC. Compute the total horizontal and vertical forces exerted on ABC by oil and water.



[15 Marks]

Sol:

For horizontal force = $F_{H_1} + F_{H_2}$

$$F_{H_1} = \text{Force on } AB = \rho g \bar{h} A_v$$

$$\bar{h} = \text{Location of C.G.} = \left(3 + 2 - \frac{4r}{3\pi} \right)$$

$$= \left(5 - \frac{4 \times 2}{3\pi} \right) = 4.15 \text{ m} \quad (\text{From top})$$

$$A_v = \frac{1}{2} \times (\pi r^2) \quad (\text{Hemispherical leads to produce semi-circle})$$

$$= \frac{1}{2} \times (\pi \times 2^2) = 6.28 \text{ m}^2$$

$$F_{H_1} = (0.9 \times 9.81) \times 4.15 \times 6.28$$

$$= 230.1 \text{ kN} (\leftarrow)$$

$$F_{H_2} = \text{Horizontal force on } BC$$

$$= (\rho g \bar{h}) \times A_v$$

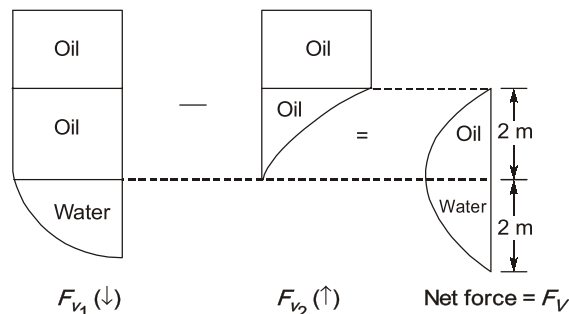
$$= (\text{Pressure of C.G.}) \times \text{Vertical projection of B.C.}$$

$$= \left(0.9 \times 9.81 \times 5 + 1 \times 9.81 \times \frac{4r}{3\pi} \right) \times 6.28$$

$$= 329.69 \text{ kN} (\leftarrow)$$

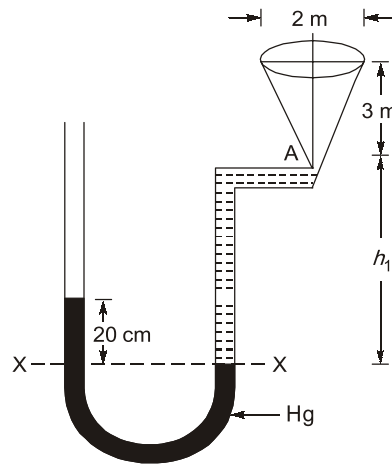
$$\text{Total horizontal force} = 230.1 + 329.69 = 559.79 \text{ kN}$$

For vertical force



$$\text{Net vertical force, } F_v = \frac{1}{4} \left(\frac{4}{3} \pi r^3 \right) (\gamma_w + 0.9 \gamma_w) = \frac{1}{3} \times \pi \times 2^3 \times \gamma_w \times 1.9 = 156.07 \text{ kN}$$

- 2.10** Figure below shows a conical vessel having its outlet at A to which a U-tube manometer is connected. The reading of the manometer as given in the figure shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.



[20 marks]

Sol:

Vessel is empty.

Difference of mercury level

$$h_2 = 20 \text{ cm}$$

Let

$$h_1 = \text{Height of water above X-X}$$

Sp. gr. of mercury,

$$S_2 = 13.6$$

Sp. gr. of water,

$$S_1 = 1.0$$

Density of mercury,

$$\rho_2 = 13.6 \times 1000 \text{ kg/m}^3$$

Density of water,

$$\rho_1 = 1000 \text{ kg/m}^3$$

Equating the pressure above datum line X-X, we have

$$\rho_2 \times g \times h_2 = \rho_1 \times g \times h_1$$

$$\Rightarrow 13.6 \times 1000 \times 9.81 \times 0.2 = 1000 \times 9.81 \times h_1$$

$$\Rightarrow h_1 = 2.72 \text{ m of water}$$

Vessel is full of water: When vessel is full of water, the pressure in the right limb will increase and mercury level in the right limb will go down. Let the distance through which mercury goes down in the right limb be, y cm as shown in figure. The mercury will rise in the left by a distance of y cm. Now the datum line is Z-Z. Equating the pressure above the datum line Z-Z.

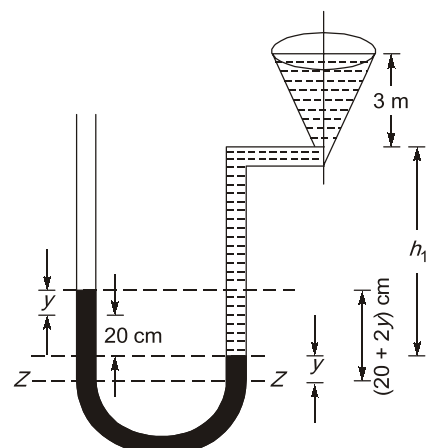
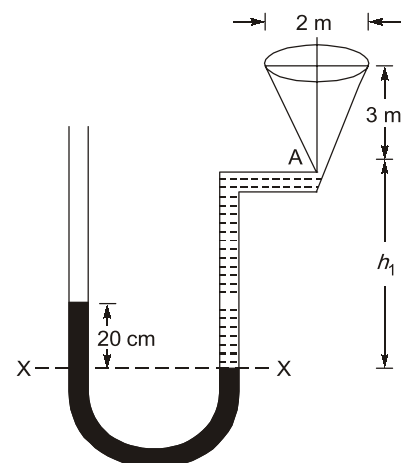
Pressure in left limb = Pressure in right limb

$$\Rightarrow 13.6 \times 1000 \times 9.81 \times \left(\frac{20 + 2y}{100} \right) = 1000 \times 9.81 \times \left(3 + h_1 + \frac{y}{100} \right)$$

$$\Rightarrow 13.6 \times \left(\frac{20 + 2y}{100} \right) = \left(3 + 2.72 + \frac{y}{100} \right)$$

$$\Rightarrow 272 + 27.2y = 5.72 \times 100 + y$$

$$\Rightarrow 26.2y = 300$$



$$\Rightarrow y = 11.45 \text{ cm}$$

The difference of mercury levels in two limbs

$$= (20 + 2y) \text{ cm of mercury}$$

$$= 20 + 2 \times 11.45 = 20 + 22.90 = 42.90 \text{ cm of mercury}$$

$$\therefore \text{Reading of manometer} = 42.90 \text{ cm}$$

3. Fluid Kinematics

Level-1

3.1 If $\phi = \frac{-A}{2\pi} \log r$, where A is a positive constant, determine ψ and plot the typical equipotential lines and stream-lines. Identify the flow pattern.

[12 Marks]

Sol:

$$\phi = -\frac{A}{2\pi} \log r$$

Above velocity potential function shows radial flow like source or sink flow.

For such flows, the radial and tangential components in terms of ϕ are given by

$$u_r = \frac{\partial \phi}{\partial r} \text{ and } u_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta}$$

$$\therefore u_r = \frac{\partial}{\partial r} \left[-\frac{A}{2\pi} \log r \right]$$

$$\Rightarrow u_r = -\frac{A}{2\pi} \cdot \frac{1}{r} \quad \dots(i)$$

$$\text{Also } u_\theta = \frac{1}{r} \frac{\partial}{\partial \theta} \left[-\frac{A}{2\pi} \log r \right] = 0$$

Also the radial and tangential component in terms of ψ are given by

$$u_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} \text{ and } u_\theta = -\frac{\partial \psi}{\partial r}$$

$$\therefore \frac{1}{r} \frac{\partial \psi}{\partial \theta} = -\frac{A}{2\pi r} \quad [\text{From (i)}]$$

$$\Rightarrow \frac{\partial \psi}{\partial \theta} = -\frac{A}{2\pi}$$

$$\Rightarrow \partial \psi = -\frac{A}{2\pi} d\theta$$

$$\Rightarrow \psi = -\frac{A}{2\pi} \theta + C_1$$

Let $\psi = 0^\circ$, when $\theta = 0^\circ$,

then

$$0 = 0 + C_1$$

$$\therefore C_1 = 0$$

Hence the equation of stream function becomes

$$\psi = -\frac{A}{2\pi} \theta$$