ESE 2027

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Main Examination

Electronics & Telecommunication Engineering

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Paper-I

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B. Singh (Ex. IES)

Director's Message

In past few years ESE Main exam has evolved as an examination designed to evaluate a candidate's subject knowledge. Studying engineering is one aspect but studying to crack prestigious ESE exam requires altogether different strategy, crystal clear concepts and rigorous practice of previous years' questions. ESE mains being conventional exam has subjective nature of questions, where an aspirant has to write elaborately - step by step with proper and well labeled diagrams and figures. This characteristic of the main exam gave me the aim and purpose to write this book. This book is an effort to cater all the difficulties being faced by students during their preparation right from conceptual clarity to answer writing approach.

MADE EASY Team has put sincere efforts in solving and preparing accurate and detailed explanation for all the previous years' questions in a coherent manner. Due emphasis is made to illustrate the ideal method and procedure of writing subjective answers. All the previous years' questions are segregated subject wise and further they have been categorised topic-wise for easy learning and helping aspirants to solve all previous years' questions of particular area at one place. This feature of the book will also help aspirants to develop understanding of important and frequently asked areas in the exam.

I would like to acknowledge the efforts of entire MADE EASY team who worked hard to solve previous years' questions with accuracy. I hope this book will stand upto the expectations of aspirants and my desire to serve the student community by providing best study material will get accomplished.

B. Singh (Ex. IES)
CMD, MADE EASY Group

ESE 2027 Main Examination

E&T Engineering — Conventional Solved Questions

Paper-I

Contents

1 Basic Electronics Engineering (EDC)

Revised Syllabus of ESE: *Basics of semiconductors; Diode/Transistor basics and characteristics; Diodes for different uses; Junction and Field Effect Transistors (BJTs, JFETs, MOSFETs); Optical sources/detectors; Basics of Opto electronics and its applications.*

1. Semiconductor Physics

1.1 Pure silicon has an electrical resistivity of 3000 Ωm. If the free carrier density in it is $1.1 \times 10^6\ m^{-3}$ and the electron mobility is three times that of hole mobility, calculate the mobility values of electrons and holes.

[15 marks : 2003]

Solution:

$$\text{Conductivity, } (\sigma) = \frac{1}{\text{resistivity}}$$

$$\text{For a pure silicon, } \sigma = (\mu_n + \mu_p)n_i e$$

Where,
μ_n = electron mobility
μ_p = hole mobility
n_i = carrier concentration
e = electron charge $\approx 1.6 \times 10^{-19}$C

Given that: $\mu_n = 3\mu_p$

So,
$$\frac{1}{3000} = (3\mu_p + \mu_p) \times 1.1 \times 10^6 \times 1.6 \times 10^{-19}$$

or
$$\mu_p = 4.7348 \times 10^8\ m^2/V\text{-sec}$$

and
$$\mu_n = 3\mu_p = 3 \times 4.7348 \times 10^8 = 1.42 \times 10^9\ m^2/V\text{-sec}$$

1.2 Explain Hall effect.

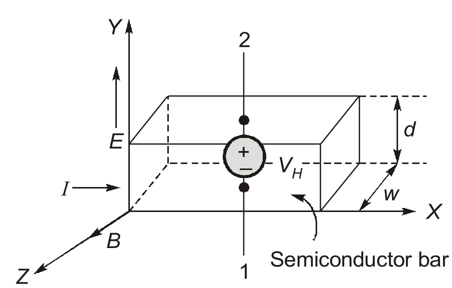
An *n*-type germanium sample is 2 mm wide and 0.2 mm thick. A current of 10 mA is passed through the sample (x-direction) and a field of 0.1 Weber/m^2 is directed perpendicular to the current flow (z-direction). The developed Hall voltage is – 1.0 mV. Calculate the Hall constant and the number of electrons/m^3.

[15 marks : 2004]

Solution:

■ Hall effect:

- If a specimen (metal or semiconductor) carrying a current I is placed in a transverse) magnetic field B, an electric field E is induced in the direction perpendicular to both I and B. This phenomenon, known as the Hall effect is used to determine whether a semiconductor is *n*-type or *p*-type and to find the carrier concentration.



⇒ As shown in the figure above, if I is in the positive X direction and B is in the positive Z direction, a force will be exerted in the negative Y direction on the current carriers. The current I may be due to holes moving from left to right or to free electrons travelling from right to left in the semiconductor specimen. Hence, independently of whether the carriers are holes or electrons, they will be forced downward toward side 1 in the figure. Hence a potential, called the Hall voltage, V_H appears between surface 1 and 2. If the polarity of V_H is positive at terminal 2 with respect to terminal 1, then the carriers must be electrons. If terminal 1 becomes charged positively with respect to terminal 2, the semiconductor must be p-type.

Give that,

$$w = 0.2\text{ cm}; \qquad d = 0.02\text{ cm}$$
$$I = 10 \times 10^{-3}\text{A} \qquad B = 0.1\text{ Wb/m}^2$$
$$V_H = 10^{-3}\text{ V}$$

We know that
$$V_H = \frac{BI}{\rho w} = \frac{BI\,R_H}{w} \qquad \left(\frac{1}{\rho} = R_H\right)$$

$$R_H = \frac{V_H w}{BI} = \frac{10^{-3} \times 0.2 \times 10^{-2}}{0.1 \times 10^{-2}} = 2 \times 10^{-3}\text{ m}^3/\text{Coulomb} = \frac{1}{nq}$$

$$2 \times 10^{-3} = \frac{1}{n \times 1.6 \times 10^{-19}}$$

$$n = 3.125 \times 10^{21}/\text{m}^3$$

1.3 A Si wafer is doped with 10^{15} phosphorus atoms cm^{-3}. Find the carrier concentration and Fermi level at room temperature (300° K), assume $N_C = 2.8 \times 10^{19}/\text{cm}^3$ at 300 K. Explain the concept of Fermi level.

[10 Marks : 2005]

Solution:

Since Si-wafer is doped with phosphorus atom, so it is a n-type extrinsic semiconductor.

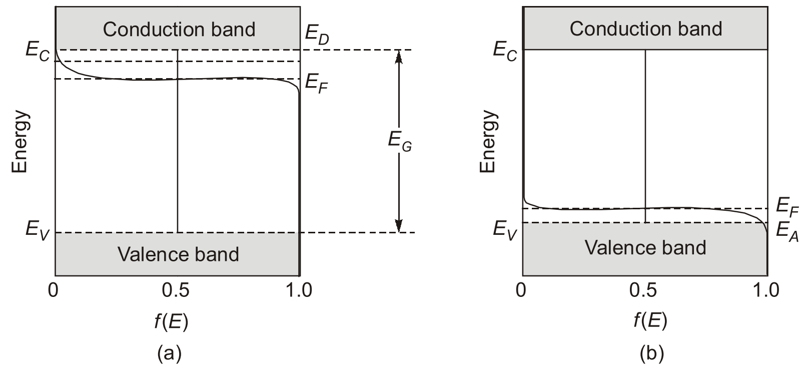
So,
$$N_D = 10^{15}\text{ atoms/cm}^3 \approx n$$
$$N_C = 2.8 \times 10^{19}\text{ atoms/cm}^3$$

According to Mass-action Law,
$$np = n_i^2 \qquad (n_i = 1.5 \times 10^{10}/\text{cm}^3 \text{ for Si at room temp.})$$

⇒ carrier concentration
$$(n_i) = \sqrt{np}$$

$$p = \frac{n_i^2}{n} = \frac{2.25 \times 10^{20}}{10^{15}} = 2.25 \times 10^5/\text{cm}^3$$

- Concept of Fermi Level (E_F) in extrinsic semiconductor:
  - ⇒ Fermi-level is a measure of probability of occupancy of the allowed energy states.
  - ⇒ In extrinsic semiconductor Fermi level moves closer to the conduction band indicating that many of the energy states in that band are now filled by donor electrons and the number of holes in the valence band are few. This concept is in respect of n-type material. While for p-type material the Fermi level is nearer to the valence band.
  - ⇒ So, the CB and VB positions will shift relatively to each other in n-type an d p-type material as shown in figure below:



Positions of Fermi level in (a) *n*-type and (b) *p*-type semiconductors

⇒ As temperature increases the density of electron increases and hence conductivity increases.

⇒ A calculation of exact position of the Fermi level in an n-type semiconductor is,

$$E_F = E_C - kT\,ln\left(\frac{N_C}{N_D}\right)$$

⇒
$$E_C - E_F = kT ln\left(\frac{N_C}{N_D}\right) \quad \text{...(i)}$$

For p-type material,

∴
$$E_F = E_V + kT ln\left(\frac{N_V}{N_A}\right)$$

⇒
$$E_F - E_V = kT ln\left(\frac{N_V}{N_A}\right) \quad \text{...(ii)}$$

Now, we have to find the Fermi level at 300° K for n-type material.
From equation (i) we have,

$$E_C - E_F = 0.03 ln\left(\frac{2.8\times10^{19}}{10^{15}}\right) \qquad \text{(At room temp. } KT \approx 0.03 \text{ eV)}$$

∴
$$(E_C - E_F) = 0.307 \text{ eV}$$

Thus E_F will be 0.307 *eV* below the bottom of the conduction band, for a donor density 10^{15} atoms/cm^3.

1.4 When a current is passed through a semiconductor and a magnetic field is applied at right angles to the direction of the current flow, it is observed that an electric field is induced in a direction mutually perpendicular to the magnetic field and the flow of current. Name this phenomenon and calculate the voltage developed.

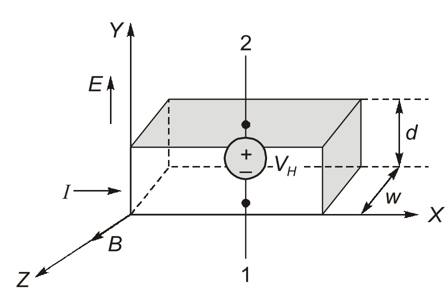
[15 Marks : 2005]

Solution:

The given phenomenon is Hall Effect.

According to this effect the statements are:

- If a specimen (metal or semiconductor) carrying a current I is placed in a transverse magnetic field B, an electric field E is induced in the direction perpendicular to both I and B. This phenomenon, known as the Hall effect, is used to determine whether a semiconductor is n-type or p-type and to find the carrier concentration.



- Consider a semiconductor specimen bar having volume charge density ρ_v (in C/m^3), width 'w', thickness 'd', cross-sectional area 'A' and developed Hall voltage is "V_H".

Since, $$E = \frac{F}{e}$$

$\therefore$ $$F = eE \quad \text{...(i)}$$

also, $$\vec{F} = q(\vec{v} \times \vec{B}) \quad \text{...(ii)}$$

In the equilibrium state,

Force on specimen due to electric field (E) = Force on specimen due to magnetic field (H)

$\Rightarrow$ $$e\,E = e\,v_d\,B \qquad \text{[where } v_d = \text{drift velocity]}$$

$\Rightarrow$ $$E = v_d\,B \quad \text{...(iii)}$$

Also, $$E = \frac{V_H}{d} \quad \text{...(iv)}$$

$\therefore$ $$V_H = B v_d d \quad \text{...(v)}$$

Drift velocity of charge carrier $= v_d = \dfrac{E}{B} = \dfrac{J}{\rho_v}$

Now from equation (v),

$$V_H = \frac{BdJ}{\rho_v} = \frac{BId}{A\rho_v} = \frac{BId}{w \times d \times \rho_v}$$

$\Rightarrow$ $$V_H = \frac{BI}{\rho_v w} \quad \text{...(vi)}$$

This equation (vi) represents the Hall voltage (V_H) developed in a semiconductor bar.

1.5 The intrinsic resistivity of Germanium at room temperature is 0.47 Ω-cm. The electron and hole mobilities at room temperature are 0.39 and 0.19 m^2/V-s respectively. Calculate the density of electrons in the intrinsic semiconductor. Also calculate the drift velocities of these charge carriers for a field of 10 kV/m.

[10 marks: 2005]

Solution:

Given that: Resistivity $\rho_i = 0.47\ \Omega$ cm

$\mu_n = 0.39$ m^2/V-sec

$\mu_p = 0.19$ m^2/V-sec

Now $$\rho_i = \frac{1}{\sigma_i} = \frac{1}{n_i q(\mu_n + \mu_p)}$$

$$0.47 \times 10^{-2} = \frac{1}{n_i \times 1.6 \times 10^{-19}(0.39 + 0.19)}$$

$$n_i = 2.29 \times 10^{21}/\text{m}^3$$

Drift velocity $v_d = \mu E$

For electron $v_d = \mu_n E = 0.39 \times 10$ km/sec

$v_d = 3.9$ km/sec

For hole $v_d = \mu_p E = 0.19 \times 10 = 1.9$ km/sec

1.6 In intrinsic GaAs, the electron and hole mobilities are 0.85 and 0.04 m^2/V-S respectively and corresponding effective masses are 0.068 m_0 and 0.5 m_0 respectively, where m_0 is the rest mass of an electron. If the energy gap of GaAs at 300° K is 1.43 eV, calculate the intrinsic carrier concentration and conductivity. ($m_0 = 9.11 \times 10^{-28}$ gm).

[15 Marks : 2006]

Solution:

Given that, $\mu_n = 0.85\ m^2/V\text{-sec}$; $\mu_p = 0.04\ m^2/V\text{-sec}$; $m_n = 0.068\ m_0$; $m_p = 0.5\ m_0$

$$E_g(300) = 1.43\ eV$$

$$m_0 = 9.11 \times 10^{-28} gm$$

Now

$$N_c = 2\left(\frac{2\pi\, m_n kT}{h^2}\right)^{3/2}$$

$$N_c = 2\left[\frac{2\times\pi\times 0.068\times 9.11\times 10^{-31}\times 1.38\times 10^{-23}\times 300}{\left(6.626\times 10^{-34}\right)^2}\right]^{3/2}$$

$$N_c = 2 \times 2.2236 \times 10^{23} = 4.447 \times 10^{23}\ /m^3$$

$$N_v = 2\left(\frac{2\pi m_p kT}{h^2}\right)^{3/2} = 2\left[\frac{2\times\pi\times 0.5\times 9.11\times 10^{-31}\times 1.38\times 10^{-23}\times 300}{\left(6.626\times 10^{-34}\right)^2}\right]^{3/2}$$

$$N_v = 2 \times 4.4335 \times 10^{24}$$

$$N_v = 8.867 \times 10^{24}\ /m^3$$

$\therefore$

$$n_i = \sqrt{N_c N_v}.\exp\left(-\frac{E_g}{2kT}\right) = \sqrt{4.447\times 8.867\times 10^{47}}\times\exp\left(\frac{-1.43}{2\times 26\times 10^{-3}}\right)$$

$$n_i = 2.26 \times 10^{12}\ /m^3$$

Conductivity

$$\sigma = n_i q(\mu_n + \mu_p) = 2.26 \times 10^{12} \times 1.6 \times 10^{-19}[0.85 + 0.04]$$

$$\sigma = 3.22 \times 10^{-7}\ S$$

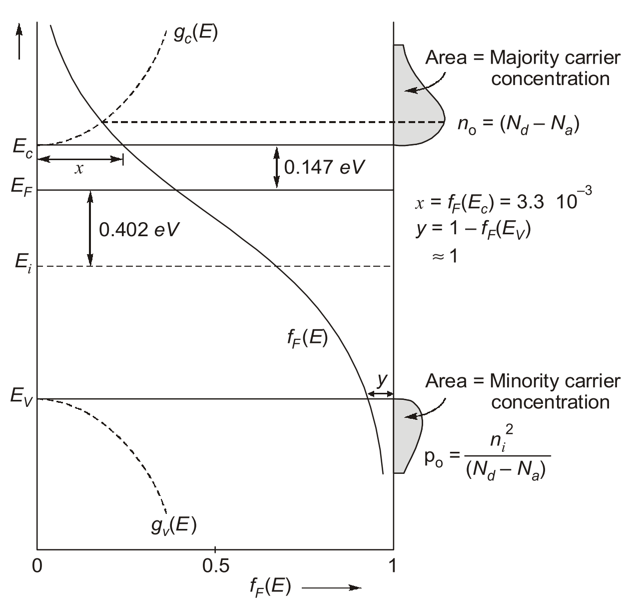
1.7 Sketch the variation of the density of states function, Fermi Dirac probability distribution function and electron and hole concentration for n-type silicon where $N_d = 10^{17}$/c.c. and $N_a = 10^{16}$/c.c. Write down the condition for charge neutrality in a compensated semiconductor at $T = 300°$ K.

In an n-type semiconductor at $T = 300°$ K, the electron concentration varies linearly from 2×10^{18} to 5×10^{17} per c.c. over a distance of 1.5 mm and the diffusion current density is 360 A/cm². Find the mobility of electrons.

[15 Marks : 2006]

Solution:

PART-1



Above diagram shows:

(a) Density of states function $g_c(E)$, $g_v(E)$ in conduction and valance band respectively.

(b) Fermi-dirac distribution function $f_F(E)$.

(c) Electron and hole concentration (n_o, p_o) for a n-type compensated semiconductor.

$\because$ $n_i(300\text{ K})$ = Intrinsic carrier concentration for Silicon $= 1.5 \times 10^{10}/\text{cm}^3$

and given $N_d = 10^{17}/\text{cm}^3, \quad N_a = 10^{16}/\text{cm}^3$

Where N_d, N_a are donor and acceptor concentrations

$$\Rightarrow \quad N_d - N_a = (10^{17} - 10^{16})/\text{cm}^3 = 9 \times 10^{16}/\text{cm}^3$$

$$\because \quad E_F - E_i \;\Delta\; KT\,ln\left(\frac{N_d - N_a}{n_j}\right) = 0.0258\,ln\left(\frac{9\times10^{16}}{1.5\times10^{10}}\right) = 0.0256 ln(6\times10^6)$$

$$E_F - E_i = 0.0258 \times 15.607 = 0.402 \text{ eV}$$

and
$$E_c - E_F = \frac{E_g}{2} - (E_F - E_{Fi}) = (0.55 - 0.402)\,\text{eV} = 0.147 \text{ eV}$$

$$\Rightarrow \quad f_F(E_c) = \frac{1}{1+\exp\left(\frac{E_c - E_F}{KT}\right)} = \frac{1}{1+\exp\left(\frac{0.147}{0.0258}\right)} = \frac{1}{1+\exp(5.69)} = 3.3\times10^3$$

Condition for charge neutrality $\Rightarrow$ positive charge = negative charge

$$\Rightarrow \quad N_d^+ + p_o = N_a^- + n_o$$

N_d^+ = Donor ion concentration

N_d^- = Acceptor ion concentration

P_o, n_o equilibrium hole, electron concentration in valance band and conduction band respectively.

PART-2

Now, in an n-type semiconductor at 300°K, diffusion electron current density = J_n.

$$\Rightarrow \quad J_n = qD_n\left|\frac{dn}{dx}\right|$$

$$\Rightarrow \quad 360\text{ A/cm}^2 = 1.6\times10^{-19}\times D_n \times\left|\frac{5\times10^{17} - 2\times10^{18}}{1.5\,mm}\right|$$

$$\Rightarrow \quad 360 = 1.6\times10^{-19}\times\frac{15\times10^{17}}{1.5\times10^{-1}}\times D_n$$

$$\Rightarrow \quad \text{Diffusion constant for electron} = D_n = \frac{360}{1.6} = 225 \text{ cm}^2/\text{sec}$$

Again by using Einstein relationship,

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$$

$$\Rightarrow \quad \mu_n = \frac{D_n}{V_T} = \frac{225\text{cm}^2/\text{sec}}{0.026 \text{ Volt}}$$

$\therefore$ mobility of electron $= \mu_n = 8653.84 \text{ cm}^2/\text{V-sec}$

1.8 **Define carrier mobility. Draw a graph showing the variation of carrier mobility in a semiconductor with increasing temperature. A 100-ohm resistor is to be made at room temperature in a rectangular silicon bar of 1 cm in length and 1 mm^2 in cross-sectional area by doping it appropriately with phosphorous atoms. If the electron mobility in silicon at room temperature be 1350 cm^2/V. sec, calculate the dopant density needed to achieve this. Neglect the insignificant contribution by the intrinsic carriers.**

[15 Marks : 2007]

Solution:

If a constant electric field E is applied to the semi conductor, as a result of this electrostatic force and electron would be accelerated and velocity would increase indefinitely with time, till it will not collide with ions. At each in elastic collision with an ion, electron loss energy and steady state condition is reached, where finite value of speed called drift speed is attained.

So drift speed v_d is proportional to $\in$.

$$v_d \propto E$$

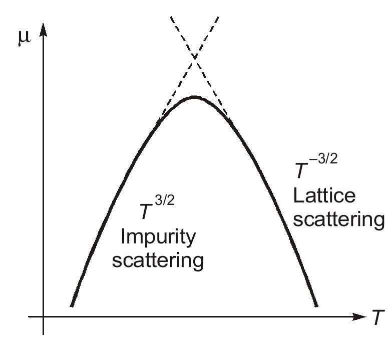
$$v_d = \mu E$$

where μ = Mobility

Mobility is defined as drift velocity per unit electric field.

$$\mu = \frac{v_d}{E}$$

Relation between Mobility with Temperature



Two types of scattering influence electron and hole mobility are

1. Lattice scattering, and
2. Impurity scattering

In Lattice scattering a carrier moving through crystal is scattered by a vibration of lattice, resulting from Temperature. Frequency of such scattering events increases as temperature increases, since thermal agitation of lattice becomes greater. Therefore we should expect the mobility to decrease with increase in temperature.

On other hand scattering from crystal defect becomes dominant mechanism at low temperature since a slowly moving carrier is likely to be scattered move strongly by an interaction with a charged iron than is a carrier with greater momentum. So impurity scattering event cause a decrease in mobility with decrease in temperature.

$$R = 100\ \Omega$$

$$l = 1\text{ cm}$$

$$\mu_n = 1350\text{ cm}^2/\text{vsec}$$

$$A = 1\text{ mm}^2 = 10^{-2}\text{ cm}^2$$

we know that

$$R = \rho\frac{l}{A}$$

$$\rho = \frac{RA}{l} = \frac{100\times 10^{-2}}{1} = 1\ \Omega\text{cm}$$

$$\sigma_N = \frac{1}{\rho} = nq\mu_n$$

$$n = \frac{1}{\rho\times q\times \mu_n} = \frac{1}{1\times 1.6\times 10^{-19}\times 1350} = 4.6\times 10^{15}/\text{cm}^3$$

For N-type

$$n \simeq N_D$$

$$N_D = 4.6\times 10^{15}/\text{cm}^3$$

1.9 A silicon bar is doped with 10^{17} arsenic atoms/cm^3. What is the equilibrium hole concentration at 300 K? Where is the Fermi level (E_F) of the sample located relative to intrinsic Fermi level (E_i)? It is known that $n_i = 1.5 \times 10^{10}$/ cm^3. [10 Marks : 2008]

Solution:

Since doped material is Arsenic i.e. pentavalent impurity, so it is an *n*-type material.

$$N_D = 10^{17} \text{ atoms/cm}^3$$
$$n_i = 1.5 \times 10^{10}/\text{cm}^3$$

According to the law of mass-action

$$np = n_i^2 = 2.25 \times 10^{20} \quad \text{...(i)}$$

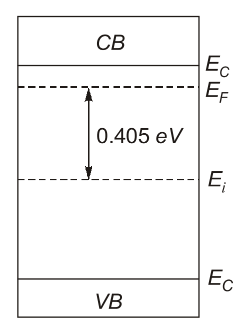
According to the principle of electrical neutrality

$$n + N_A = p + N_D$$
$$\Rightarrow \quad n = p + N_D = p + 10^{17}$$
$$\therefore \quad n \simeq 10^{17} \quad [\text{as } N_D >> p]$$

Therefore $$p = \frac{2.25 \times 10^{20}}{10^{17}} = 2.25 \times 10^3 \text{ atoms/cm}^3$$

Fermi level energy for *n*-type material is given by relation

$$n = n_i e^{[E_F - E_i]}/kT$$
$$\frac{n}{n_i} = e^{[E_F - E_i]}/kT$$
$$\Rightarrow \quad E_F - E_i = KT\,ln\left(\frac{n}{n_i}\right) = 8.62 \times 10^{-5} \times 300\; ln\left(\frac{10^{17}}{1.5 \times 10^{10}}\right)$$
$$\Rightarrow \quad E_F - E_i = 0.0258\; ln\left(\frac{10^7}{1.5}\right)$$
$$\Rightarrow \quad E_F - E_i = 0.405 \text{ eV}$$

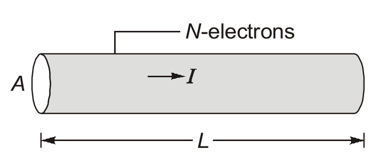


1.10 Derive an expression for current density in an n-type semiconductor in terms of drift velocity. Show that the conductivity is given by $\sigma = ne\mu_n$. [15 Marks : 2008]

Solution:

- **Current density**

- If *N* electrons are contained in a length *L* of a conductor as shown in figure below. If *T* is time taken to traverse distance *L*, the total number of electrons passing through any cross section of wire in *T* per unit time is *N*/*T*.



(A = cross-sectional area)

Therefore, $$I = \frac{Nq}{T} = \frac{Nqv}{L} \quad \text{...(i)}$$

$\therefore$ Current density $$(J) = \frac{I}{A} = \frac{Nqv}{LA} \quad \text{...(ii)}$$

(unit of J = amp/m^2)

Since, $\frac{N}{LA} = n$ (electron concentration in electrons per cubic meter)

∴ $J = nqv = nev = \rho v$ ...(iii)

where $\rho \equiv ne$ is the **charge density** in Coulombs per cubic meter and v in meters per second.

- The conductivity of a material can be related to the number of charge carriers present in the materials. Now, combining equations (i) and (iii) we get,

$$J = nqv = nq\mu E$$

$\Rightarrow$ $J = \sigma E$ ...(iv)

where, $\sigma = nq\mu$ ...(v)

where σ is conductivity in (ohm-meter)$^{-1}$

1.11 Derive an expression for current density in an n-type semiconductor in terms of drift velocity. Show that the conductivity is given by $\sigma = ne\mu_n$ [15 marks: 2008]

Solution:

Let force F on the particle when an electric field E is applied is given by

$$F = qE = eE$$

We know that $F = ma$

so $ma = eE$

So Acceleration $a = \frac{eE}{m}$

Because of collision of electron during motion, electron, will not get accelerated indefinitely, if τ is the relaxation time, average velocity of electron known as drift velocity given by

$$v_d = a \times \tau = \frac{aE}{m}\tau$$

Let current I flowing through conductor on application of E with corresponding drift velocity v_d. In time dt, electron will travel $v_d dt$ and number of electron crossing cross-section A will be $Av_d dt$.

Total charge flowing through section $dq = env_d A\, dt$

So $I = \frac{dq}{dt} = env_d A$

Now current density $J = \frac{I}{A}$

By ohm's law $J = \sigma E$

and $v_d = \mu E$

$$\sigma E = env_d$$

$$\sigma E = en\mu E$$

$$\sigma = en\mu$$

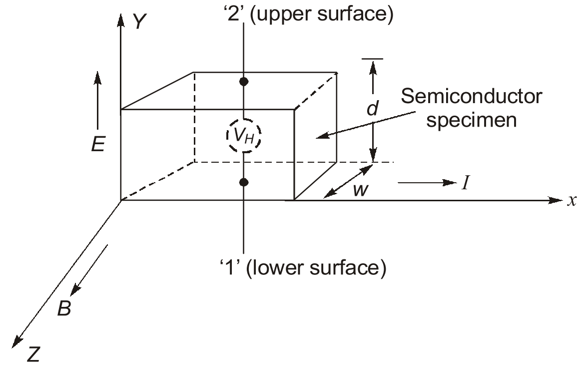
1.12 Describe the Hall effect in a semiconductor bar specimen. Derive the expression for the Hall voltage. [15 marks: 2009]

Solution:

■ Hall Effect:

- The **Hall effect** has played a decisive role in revealing the mechanism of conduction in semiconductors.
- "If a specimen (metal or semiconductor) carrying a current 'I' is placed in a **transverse magnetic field** 'B', an electric field 'E' is induced in the direction perpendicular to both I and B. This phenomenon is known as the **"Hall effect"**.
- In figure 'I', is in positive X-direction and B is in positive Z-direction a force will be exerted in the negative Y-direction on the current carriers.

- The current (I) may be due to holes (if specimen is p-type) moving from left to right or to free electrons (if specimen is n-type) moving from right to left in the specimen.
- Hence independently of whether the carriers are holes or electrons, they will be forced downward **toward side '1'.**
- If the semiconductor is **n-type material**, then all free electrons will accumulate on **side '1'** and so the lower surface becomes negatively charged with respect to **side '2'**. Hence a potential, called the **Hall voltage** (V_H) appears between surfaces '1' and '2'.
- If the polarity of **"V_H"** is positive at terminal '2', then as explained above, the carrier must be electrons and so the specimen is **n-type.**
  On the other hand, if terminal '1' becomes charged positively with respect to terminal '2', the semiconductor bar must be **p-type.**



- Consider a semiconductor specimen bar having volume charge density ρ_v (in C/m^3), width 'w', thickness 'd', cross-sectional area 'A' and developed Hall voltage is "V_H".

Since, $$E = \frac{F}{e}$$

∴ $$F = eE \quad \text{...(i)}$$

also, $$\vec{F} = q(\vec{v}\times\vec{B}) \quad \text{...(ii)}$$

In the equilibrium state,

Force on specimen due to electric field (E) = Force on specimen due to magnetic field (H)

⇒ $$eE = e\,v_d\,B \quad \text{[where } v_d = \text{drift velocity]}$$

⇒ $$E = v_d B \quad \text{...(iii)}$$

Also, $$E = \frac{V_H}{d} \quad \text{...(iv)}$$

∴ $$V_H = Bv_d d \quad \text{...(v)}$$

drift velocity of charge carrier = $v_d = \dfrac{E}{B} = \dfrac{J}{\rho_v}$

Now from equation (v), $$V_H = \frac{BdJ}{\rho_v} = \frac{BId}{A\rho_v} = \frac{BId}{w\times d\times \rho_v}$$

⇒ $$V_H = \frac{BI}{\rho_v w} \quad \text{...(vi)}$$

This equation (vi) is the required expression for the Hall voltage.

1.13 The mobilities of electron and hole in silicon sample are 0.125 m^2/V-s and 0.048 m^2/V-s respectively. Determine the conductivity of intrinsic silicon at 27°C, if the intrinsic carrier concentration is 1.6 × 10^{16} atoms/m^3. When it is doped with 10^{23} phosphorus atoms/m^3, determine the equilibrium hole concentration, conductivity and position of the Fermi level relative to the intrinsic level.

[15 Marks : 2009]

Solution:

Given that,

$$\mu_n = 0.125 \text{ m}^2/\text{V-sec}$$
$$\mu_p = 0.048 \text{ m}^2/\text{V-sec}$$
$$n_i = 1.6 \times 10^{16} \text{ atoms/m}^3$$
$$N_D = 10^{23} \text{ atoms/m}^3$$

1. Hole concentration

By Mass action law

$$p = \frac{n_i^2}{N_D} = \frac{(1.6 \times 10^{16})^2}{10^{23}}$$
$$p = 2.56 \times 10^9 \text{ atoms/m}^3$$

2. Conductivity

Intrinsic conductivity

$$\sigma_i = qn_i(\mu_n + \mu_p) = 1.6 \times 10^{-19} \times 1.6 \times 10^{16} \times (0.125 + 0.048)$$
$$\sigma_i = 4.43 \times 10^{-4} \text{ S}$$

Conductivity after doping

$$\sigma_N = nq\mu_n = 1.6 \times 10^{-19} \times 10^{23} \times 0.125$$
$$\sigma_N = 2000\ S$$

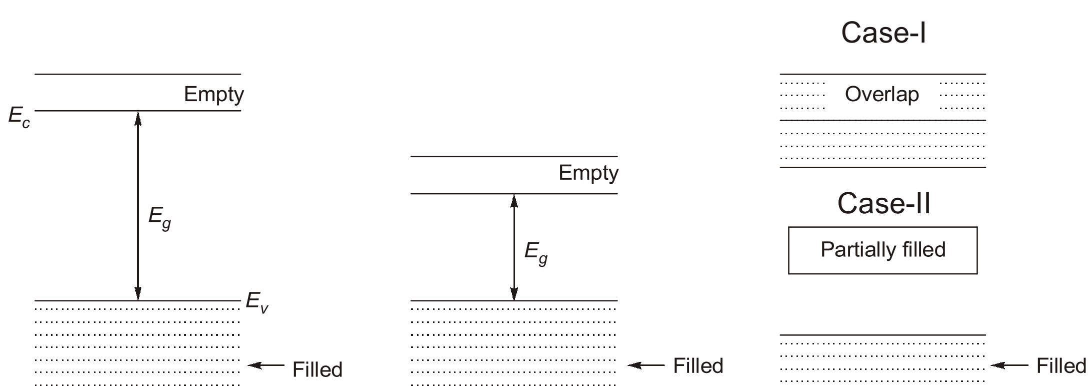
3. We know that

$$E_F - E_i = kT\,ln\left(\frac{N_D}{n_i}\right) = 0.41\ eV.\text{(upward shift w.r.t. intrinsic fermi level)}$$

1.14 Sketch the electron distribution in insulator, intrinsic semiconductor and metal at 0 K.

[10 marks : 2009]

Solution:



(a) Insulator (b) Semiconductor (c) Metal

Typical electron distribution at 0 K in metal, semiconductor, insulator.

- In insulator all the electrons are in valance band and conduction band is empty.
- In semiconductor at 0 K conduction band is empty and valance band is completely filled.
- In metal even at 0 K there are electrons in valance band as well as in conduction band.

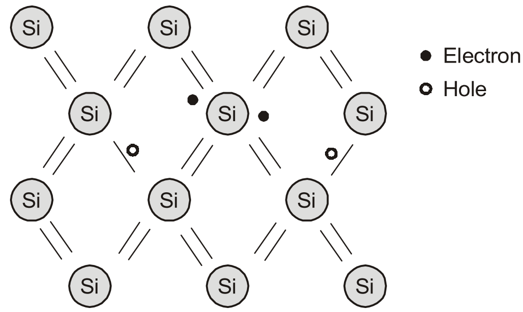
  So, either the bands overlap or conduction band is partially filled.

1.15 Sketch the covalent bonding of Si atoms in an intrinsic Si crystal.
Illustrate with sketches the formation of bonding in presence of donor and acceptor atoms. Sketch the energy band diagram, indicating the position of donor and acceptor levels at 0 K. Explain how the semiconductor behaves as n-type (in case of donor) and p-type (in case of acceptor) at a finite room temperature (T > 0 K)

[10 marks: 2010]

Solution:

INTRINSIC *Si* CRYSTAL



EXTRINSIC MATERIAL

n-type: An impurity from column *V* of the periodic table (*P*, *As*, and *Sb*) introduces an energy level very near the conduction band in Ge or Si. This level is filled with electrons at 0 K, and very little thermal energy is required to excite these electrons to the conduction band. Thus at about 50-100 K virtually all of the electrons in the impurity level are "*donated*" to the conduction band. Such an impurity level is called a donor level and the column *V* impurities in Ge or Si are called donor impurities. The material doped with donor impurities can have a considerable concentration of electrons in the conduction band, even when the temperature is too low for the intrinsic electron hole pair concentration to be appreciable Thus semiconductors doped with a significant number of donor atoms will have $n_0 >> (n_i, p_0)$ at room temperature. This is *n-type material*.

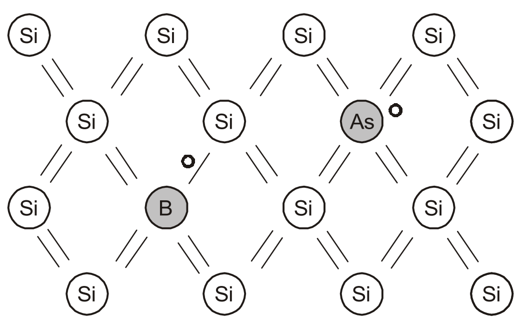


Fig. Donor & Acceptor atoms in covalent bonding model of Si crystal.

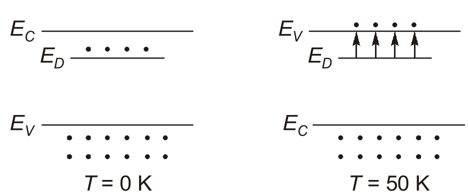


Fig. Energy band model of donor type

P-type: Atoms from column III (*B*, *Al*, *Ga*, and *In*) introduce impurity levels in Ge or Si near the valence band. These levels are empty of electrons at 0 K. At low temperatures, enough thermal energy is available to excite electrons from the valence band into the impurity level, leaving behind holes in the valence band. Since this type of impurity level "accepts" electrons from the valence band, it is called an *acceptor level*, and the column III impurities are acceptor impurities in Ge and Si. As doping with acceptor impurities can create a semiconductor with a hole concentration p_0 much greater than the conduction band electron concentration n_0 (this is *p-type* material).

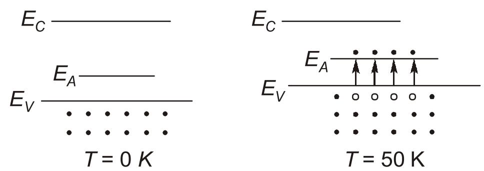


Fig. Energy band model of acceptor type

An '*As*' atom (column V) in the Si lattice has the four necessary valence electrons to complete the covalent bonds with the neighbouring Si atoms, plus one extra electron. This fifth electron does not fit into the bonding structure of the lattice and is therefore loosely bound to the '*As*' atom. A small amount of thermal energy enables this extra electron to overcome its Coulombic binding to the impurity atom and be donated to the lattice as a hole. Thus it is free to participate in current conduction. This process is a qualitative model of the excitation of electrons out of a donor level and into the conduction band. Similarly, the column III impurity '*B*' has only three valence electrons to contribute to the covalent bonding, thereby leaving one bond incomplete. With a small amount of thermal energy, this incomplete bond can be transferred to other atoms as the bonding electrons exchange positions.

1.16 Write down the expression for Fermi-Dirac distribution function. Explain the meaning of each parameter. Show that the probability that a state ΔE above the Fermi level, E_F is filled equals the probability that a state below E_F by the same amount ΔE is empty.

[10 marks: 2010]

Solution:

At thermal equilibrium

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}} \qquad k = \text{Boltzmann's constant}$$

E_F = Fermi level

The function $f(E)$, is Fermi-Dirac distribution function, gives the probability that an available energy state at E will occupied by an electron at absolute temperature T.

For an energy equal to Fermi level E_F, the occupation probability is

$$f(E_F) = \frac{1}{1+1} = \frac{1}{2}$$

Thus an energy state at Fermi level has a probability of 1/2 of being occupied by an electron.

$f(E)$ at 0 K every available energy state up to E_F is filled with electrons, and all states above E_F are empty.

At temperatures higher than 0 K, some probability exists for states above the Fermi level to be filled. *The probability $f(E_F + \Delta E)$ that a state ΔE above E_F is filled is the same as the probability $[1 - f(E_F - \Delta E)]$ that a state ΔE below E_F is empty.*

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

Let ΔE is energy state above E_F

$$E = E_F + \Delta E$$

$$\Rightarrow \qquad E - E_F = \Delta E$$

$$f(E) = \frac{1}{1+e^{+\frac{\Delta E}{kT}}}$$

Let ΔE is energy state below E_F

$$E = E_F - \Delta E$$

$$\Rightarrow \qquad E - E_F = -\Delta E$$

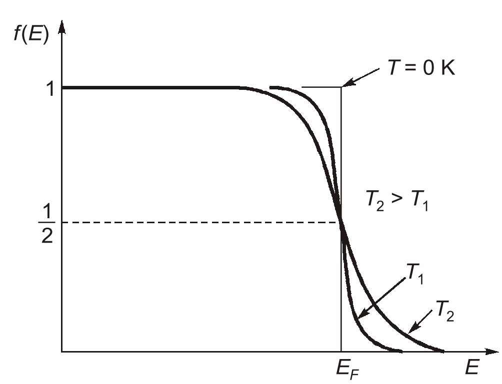


Fig. The Fermi-Dirac distribution function

$$f'(E) = \frac{1}{1+e^{\frac{-\Delta E}{kT}}}$$

The probability that $(E_F - \Delta E)$ is empty

$$P = 1 - f'(E)$$

$$\Rightarrow \quad P = 1 - \frac{1}{1+e^{-\Delta E/kT}} = \frac{1}{1+e^{\Delta E/kT}}$$

$$\therefore \quad P = f(E)$$

Hence proved.

1.17 Obtain an expression for the intrinsic Fermi level (E_i) of a semiconductor with respect to the conduction band edge. Estimate the shift of E_i from the middle of the band gap ($E_g/2$) at 300 K for InSb. Assume that the relevant ratio of electron to hole effective mass for InSb is 0.014.

[15 marks : 2010]

Solution:

Part:1

Number of electrons $\quad n = N_C e^{-(E_C - E_F)/kT}$

Number of holes $\quad p = N_V e^{-(E_F - E_V)/kT}$

for intrinsic semiconductor, $\quad n = p$

$$N_C e^{-(E_C - E_F)/kT} = N_V e^{-(E_F - E_V)/kT}$$

$$\frac{N_C}{N_V} = e^{\left[(E_C + E_V) - 2E_F\right]/kT}$$

$$ln\left(\frac{N_C}{N_V}\right) = \frac{E_C + E_V - 2E_F}{kT}$$

$$E_F = \frac{E_C + E_V}{2} - \frac{kT}{2} ln\left(\frac{N_C}{N_V}\right) = \frac{E_C + E_C - E_g}{2} - \frac{kT}{2} ln\left(\frac{N_C}{N_V}\right)$$

$$(\because E_C - E_V = E_g)$$

$$E_F = E_C - \frac{E_g}{2} - \frac{kT}{2} ln\left(\frac{N_C}{N_V}\right)$$

for $\quad m_n = m_p$

so $\quad N_C = N_V$

So we can conclude that $\quad E_F = E_C - \frac{E_g}{2}$

Part:2

$$E_F' = E_F - \frac{kT}{2} ln\left(\frac{N_c}{N_v}\right)$$

$$N_C = 2\left(\frac{2\pi kT m_n}{h^2}\right)^{\frac{3}{2}} \quad ; \quad N_V = 2\left(\frac{2\pi kT m_p}{h^2}\right)^{\frac{3}{2}}$$

$$\frac{N_C}{N_V} = \left(\frac{m_n}{m_p}\right)^{\frac{3}{2}}$$

Given that $\quad \frac{m_n}{m_p} = 0.014$

$$\frac{N_C}{N_V} = (0.014)^{3/2} = 0.0017$$