

ESE 2027

Main Examination

UPSC ENGINEERING SERVICES EXAMINATION

Topicwise
**Conventional
Practice Book**

Electronics & Telecommunication Engineering

PAPER-I





MADE EASY Publications Pvt. Ltd.

Corporate Office: 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016

E-mail: infomep@madeeasy.in

Contact: 9021300500

Visit us at: www.madeeasypublications.org

**ESE Main Examination • Conventional Practice Book :
Electronics & Telecommunication Engineering PAPER-I**

© Copyright, by MADE EASY Publications Pvt. Ltd.

All rights are reserved. No part of this publication may be reproduced, stored in or introduced into a retrieval system, or transmitted in any form or by any means (electronic, mechanical, photo-copying, recording or otherwise), without the prior written permission of the above mentioned publisher of this book.

First Edition: 2023

Reprint: 2024

Reprint: 2025

Reprint: 2026

ESE 2027 Main Examination

Conventional Practice Book

Electronics & Telecommunication Engineering

PAPER-I

CONTENTS

Sl.	TOPIC	PAGE No.	Sl.	TOPIC	PAGE No.
1.	Basic Electronics Engineering	1-69			
1.	Basics of Semiconductors	1	4.	Two Port Network.....	184
2.	PN Junction Diode	14	5.	Steady State Analysis and Resonance.....	192
3.	Bipolar Junctions Transistor.....	34	6.	Graph Theory and Magnetic Coupling	206
4.	Field Effect Transistors.....	45	7.	Filters and locus diagram.....	216
5.	Basics of Opto Electronic Devices.....	62			
2.	Material Science	70-111	5.	Analog Circuits	221-276
1.	Crystallography	70	1.	Operational Amplifiers	221
2.	Dielectric Properties of Materials.....	77	2.	Diodes.....	231
3.	Magnetic Properties of Materials.....	89	3.	Bipolar Junction Transistor.....	245
4.	Conductive Materials	99	4.	MOSFET	256
5.	Semiconductor Materials.....	105	5.	Multistage and Feedback Amplifiers	266
6.	Nanomaterials.....	108			
3.	Electronic Measurement & Instrumentation	112-159	6.	Digital Circuits	277-332
1.	Basics of Measurement and Error Analysis.....	112	1.	Basics of Digital Circuits	277
2.	Electromechanical Indicating Instruments	116	2.	Combinational Circuits.....	284
3.	Bridge Measurements.....	129	3.	Sequential Circuits	295
4.	Cathode Ray Oscilloscope (CRO).....	136	4.	Memories & Programmable Logical Devices.....	306
5.	Digital systems for measurement.....	141	5.	Logic Families.....	315
6.	Transducers.....	148	6.	ADCs / DACs	325
4.	Network Theory	160-220	7.	Basic Electrical Engineering	333-376
1.	Basic of Network Analysis	160	1.	Electromagnetic Induction and Magnetic Circuit.....	333
2.	Network Theorem.....	166	2.	Transformers.....	334
3.	Transient Analysis.....	174	3.	Induction Machines.....	344
			4.	DC Machines.....	354
			5.	Synchronous Machines	366
			6.	Basics of Power Generation	374



1

Basic Electronics Engineering

1. Basics of Semiconductors

Level-1

- 1.1 Calculate the concentration of holes and electrons in a sample of Germanium at 300 K which has a donor concentration of 2×10^{14} and acceptor concentration of 3×10^{14} atoms/cm³. Also calculate conductivity.

(12 Marks)

Solution:

Given, $N_D = 2 \times 10^{14} / \text{cm}^3$

$$N_A = 3 \times 10^{14} / \text{cm}^3$$

Since, $N_A > N_D$, so its a p-type SC

$$\text{Net impurity concentration} = N_A - N_D = 10^{14} / \text{cm}^3$$

$$\text{For Germanium, } n_i = 2.5 \times 10^{13} / \text{cm}^3$$

Any semiconductor is electrically neutral i.e. net negative charge = net positive charge

$$n + N_A = p + N_D; \quad \text{Electrical neutrality equation}$$

From mass action law,

$$np = n_i^2$$

$$n = \frac{n_i^2}{p}$$

$$\therefore \frac{n_i^2}{p} + N_A = p + N_D$$

$$\Rightarrow p^2 - (N_A - N_D)p - n_i^2 = 0$$

$$\Rightarrow p = \frac{(N_A - N_D) \pm \sqrt{(N_A - N_D)^2 + 4n_i^2}}{2}$$

$$\therefore p = \frac{10^{14} \pm \sqrt{(10^{14})^2 + 4(2.5 \times 10^{13})^2}}{2}$$

$$\Rightarrow p = 1.059 \times 10^{14} / \text{cm}^3$$

$$\Rightarrow n = \frac{n_i^2}{p} = \frac{(2.5 \times 10^{13})^2}{1.059 \times 10^{14}} = 5.9 \times 10^{12} / \text{cm}^3$$

$$\begin{aligned}
 \sigma &= ne\mu_n + pe\mu_p \\
 &= 5.9 \times 10^{12} \times 1.6 \times 10^{-19} \times 3800 + 1.059 \times 10^{14} \times 1.6 \times 10^{-19} \times 1800 \\
 &= 0.034 \, \Omega \text{ cm}^{-1}
 \end{aligned}$$

[**Note** : Compensation doping is a term used to describe the doping of a semiconductor with both donors and acceptors to control the properties. For example, a p-type SC doped with N_A acceptors can be converted to an n-type semiconductor by simply adding donors until the concentration N_D exceeds N_A . The effect of donors compensates for the effect of acceptors and vice versa].

- 1.2 In intrinsic GaAs, the electron and hole mobilities are 0.85 and 0.04 $\text{m}^2/\text{V}\cdot\text{sec}$ respectively and corresponding effective masses are 0.068 m_0 and 0.5 m_0 . If energy gap of GaAs at 300 K is 1.43 eV, calculate the intrinsic carrier concentration and conductivity.

(12 Marks)

Solution:

For an intrinsic GaAs,

$$\mu_n = 0.85 \, \text{m}^2/\text{V}\cdot\text{sec}, \mu_p = 0.04 \, \text{m}^2/\text{V}\cdot\text{sec}, m_n = 0.068 m_0$$

$$m_p = 0.5 m_0, E_G = 1.43 \, \text{eV @ 300 K}; n_i = ?, \sigma = ?$$

Now,

$$\begin{aligned}
 N_C &= 2 \left(\frac{2\pi m_n \bar{k}T}{h^2} \right)^{3/2} \\
 &= 2 \left(\frac{2\pi \times 0.068 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.63 \times 10^{-34})^2} \right)^{3/2} \\
 &= 2 \times (3.66 \times 10^{15})^{3/2} = 4.43 \times 10^{23}/\text{m}^3
 \end{aligned}$$

$$\begin{aligned}
 N_V &= 2 \left(\frac{2\pi m_p \bar{k}T}{h^2} \right)^{3/2} \\
 &= 2 \left[\frac{2\pi \times 0.5 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.63 \times 10^{-34})^2} \right]^{3/2} \\
 &= 8.83 \times 10^{24} / \text{m}^3
 \end{aligned}$$

$$n_i^2 = N_C N_V e^{-E_G/kT}$$

$$n_i = \sqrt{N_C N_V} e^{-E_G/2kT}$$

$$= \sqrt{4.43 \times 10^{23} \times 8.83 \times 10^{24}} e^{-\frac{1.43}{2 \times 0.026}}$$

$$n_i = 2.25 \times 10^{12}/\text{m}^3$$

$$\sigma_i = n_i (\mu_n + \mu_p) e$$

$$= 2.25 \times 10^{12} \times (0.85 + 0.04) \times 1.6 \times 10^{-19}$$

$$= 3.2 \times 10^{-7} \, \Omega \text{ m}^{-1}$$

- 1.3 If E_1 and E_2 are the two energy levels below and above E_F but equidistant from E_F , show that the probability of energy level E_1 being empty is same as the level E_2 being filled.

(10 Marks)

Solution:

Fermi-dirac function;

$$f(E) = \frac{1}{1 + e^{\frac{(E-E_F)}{kT}}}$$

Probability of E_2 being filled;

$$f(E_2) = \frac{1}{1 + e^{\frac{(E_2-E_F)}{kT}}}$$

$\therefore$

$$f(E_2) = \frac{1}{1 + e^{\frac{\Delta E}{kT}}} \quad \dots(i)$$

Probability of E_1 being empty,

$$\begin{aligned} 1 - f(E_1) &= 1 - \frac{1}{1 + e^{\frac{(E_1-E_F)}{kT}}} = 1 - \frac{1}{1 + e^{\frac{-\Delta E}{kT}}} \\ &= \frac{e^{\frac{\Delta E}{kT}}}{1 + e^{\frac{\Delta E}{kT}}} = \frac{1}{1 + e^{\frac{-\Delta E}{kT}}} \quad \dots(ii) \end{aligned}$$

Hence, from (i) and (ii)

$$f(E_2) = 1 - f(E_1)$$

or $f(E_2) + f(E_1) = 1$; where E_1 and E_2 are equidistant from E_F .

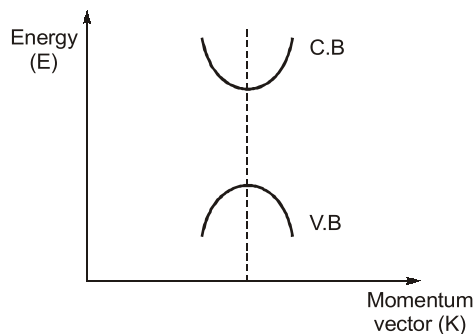
1.4 With the help of E-k diagram explain the difference between direct and indirect band semi conductors. (12 Marks)

Solution:

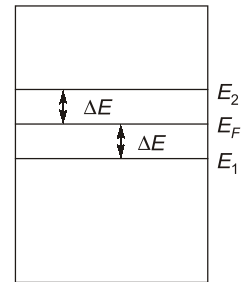
Direct Bandgap semiconductor:

- In a direct band gap semiconductor, an electron in conduction band can fall to an empty state in valence band giving off the energy difference as photon of light.

E-k diagram:

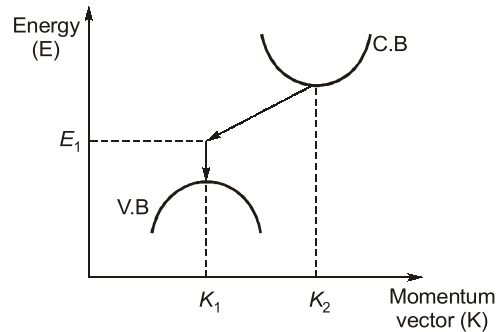


- These are the materials for which lowest energy state of conduction band and highest energy state for valence band occurs for same momentum.
- In these type of materials, recombination occurs without help of extra agent.



Indirect Bandgap semiconductor:

An electron in conduction band of an indirect bandgap semiconductor cannot fall directly to valence band but it needs to undergo a momentum change.

E-k diagram:

- These are the materials in which higher most energy state of valence band and lower most energy state of conduction band occurs for different values of momentum. Probability of recombination is very less.
- So, an external agent like gold (A_n) is required.
- Energy is released in the form of heat.

1.5 A hypothetical semiconductor has an intrinsic carrier concentration of $1.0 \times 10^{10}/\text{cm}^3$ at 300 K, it has conduction and valence band effective density of states N_C and N_V , both equal to $10^{19}/\text{cm}^3$.

- What is the energy band gap, E_g ? Assume, $KT = 0.026 \text{ eV}$
- If the semiconductor is doped with $N_d = 1 \times 10^{16} \text{ donors/cm}^3$, what are the equilibrium electron and hole concentrations at 300 K?
- If the same piece of semiconductor, already having $N_d = 1 \times 10^{16} \text{ donor/cm}^3$, is also doped with $N_a = 2 \times 10^{16} \text{ acceptors/cm}^3$, what are the new equilibrium electron and hole concentrations at 300 K?
- Consistent with your answer to part (iii), what is the Fermi level position with respect to the intrinsic fermi level, $E_F - E_i$?

(3 + 3 + 3 + 3 Marks)

Solution:

Given, intrinsic carrier concentration,

$$n_i = 1.0 \times 10^{10}/\text{cm}^3$$

temperature, $T = 300 \text{ K}$

effective density of states in conduction band,

$$N_C = 10^{19}/\text{cm}^3,$$

effective density of states in valence band,

$$N_V = 10^{19}/\text{cm}^3$$

(i) We know that,

intrinsic carrier concentration,

$$n_i = \sqrt{N_V \times N_C} e^{-E_g/2KT}$$

$$10^{10} = \sqrt{10^{19} \times 10^{19}} e^{-E_g/2 \times 0.026}$$

$$10^{10} = 10^{19} e^{-E_g/2 \times 0.026}$$

$$10^{-9} = e^{-E_g/2 \times 0.026}$$

by taking 'ln' on both sides

$$\ln(10^{-9}) = \frac{-E_g}{2 \times 0.026}$$

$$-20.723 = \frac{-E_g}{2 \times 0.026}$$

$$\therefore E_g \simeq 1.08 \text{ eV}$$

(ii) Given, donor dopant concentration,

$$N_d = 10^{16} \text{ atoms/cm}^3$$

In a doped (donor) semiconductor, the equilibrium electron concentration, $n_0 \simeq N_d$

$$\therefore n_0 \simeq 10^{16} \text{ atoms/cm}^3$$

$$\therefore \text{equilibrium hole concentration, } p_0 = \frac{n_i^2}{n_0}$$

$$\therefore p_0 = \frac{(10^{10})^2}{10^{16}} = 10^4 \text{ atoms/cm}^3$$

(iii) Given, donor dopant concentration,

$$N_d = 10^{16}/\text{cm}^3$$

acceptor dopant concentration,

$$N_a = 2 \times 10^{16}/\text{cm}^3$$

since,

$$N_a > N_d \text{ but not } N_a \gg N_d$$

$\therefore$ The equilibrium hole concentration,

$$p_0 = N_a - N_d = 2 \times 10^{16} - 10^{16} = 10^{16}/\text{cm}^3$$

the equilibrium electron concentration,

$$n_0 = \frac{n_i^2}{p_0}$$

$$n_0 = \frac{(10^{10})^2}{10^{16}} = 10^4/\text{cm}^3$$

(iv) From part (iii),

the equilibrium hole concentration,

$$p_0 = 10^{16}/\text{cm}^3$$

but,

$$p_0 = n_i e^{(E_i - E_F)/KT}$$

$$10^{16} = 10^{10} e^{(E_i - E_F)/0.026}$$

$$10^6 = e^{(E_i - E_F)/0.026}$$

$$\therefore E_i - E_F = 0.026 \ln(10^6)$$

$$\therefore E_F - E_i = -0.36 \text{ eV}$$

1.6 The energy band gap of silicon (Si) depends on the temperature as follows:

$$E_g = 1.17 \text{ eV} - 4.73 \times 10^{-4} \frac{T^2}{T + 636}$$

If the intrinsic carrier concentration of Si at $T = 300 \text{ K}$ is $1.05 \times 10^{10} \text{ cm}^{-3}$. Find the concentration of electrons in the conduction band of intrinsic (undoped) Si at temperature $T = 77 \text{ K}$? (Assume, $KT = 0.026 \text{ eV}$, $E_g(300 \text{ K}) = 1.12 \text{ eV}$)

(12 Marks)

Solution:

Given that,

Energy band gap of silicon, $E_g = 1.17 \text{ eV} - 4.73 \times 10^{-4} \frac{T^2}{T + 636}$

The intrinsic carrier concentration,

$$\therefore n_i^2 \propto T^3 e^{-E_g/KT}$$

$$n_i \propto T^{3/2} e^{-\frac{E_g}{2KT}}$$

Let at temperature, $T_1 = 300 \text{ K}$,

$$n_i(T_1) \propto (T_1)^{3/2} e^{-\frac{E_g}{2KT_1}} \quad \dots(i)$$

Let at temperature, $T_2 = 77 \text{ K}$

$$n_i(T_2) \propto (T_2)^{3/2} e^{-\frac{E_g}{2KT_2}} \quad \dots(ii)$$

dividing equation (ii) with equation (i),

$$\frac{n_i(T_2)}{n_i(T_1)} = \left(\frac{T_2}{T_1} \right)^{3/2} e^{-\left[\frac{E_g(T_2)}{2KT_2} - \frac{E_g(T_1)}{2KT_1} \right]}$$

$$\therefore n_i(77 \text{ K}) = n_i(300 \text{ K}) \left(\frac{77}{300} \right)^{3/2} e^{-\left[\frac{E_g(T_2)}{2KT_2} - \frac{E_g(T_1)}{2KT_1} \right]}$$

where,

$$E_g(300 \text{ K}) = E_g(T_1)$$

$$KT_1 = 0.026 \text{ eV}$$

$$KT_2 = \frac{T_2}{11600} = \frac{77}{11600} = 0.0066 \text{ eV}$$

$$E_g(T_2) = E_g(77 \text{ K}) = 1.17 \text{ eV} - 4.73 \times 10^{-4} \frac{(77)^2}{77 + 636}$$

$$E_g(77 \text{ K}) = 1.16 \text{ eV}$$

$$n_i(77 \text{ K}) = 1.05 \times 10^{10} \times \left(\frac{77}{300} \right)^{3/2} e^{-\left[\frac{1.16}{0.0132} - \frac{1.12}{0.052} \right]}$$

$$n_i(77 \text{ K}) = 2.11 \times 10^{-20} \text{ cm}^{-3}$$

1.7 The bandgap of GaAs and AlAs are 1.43 eV and 2.16 eV respectively. Assuming the bandgap of $\text{Al}_x \text{Ga}_{1-x} \text{As}$ to vary linearly with x between the two extreme values, find the value of x that would result in the emission of 680 nm from $\text{Al}_x \text{Ga}_{1-x} \text{As}$.

(10 Marks)

Solution:

$$E_G(\text{GaAs}) = 1.43 \text{ eV}$$

$$E_G(\text{AlAs}) = 2.16 \text{ eV}$$

$$\text{Al}_x \text{Ga}_{1-x} \text{As}; x \text{ is mole fraction}$$

x can vary from 0 to 1. By changing x , bandgap of compound semiconductor can be varied. Hence, it is called bandgap engineered semiconductor.

When

$$x = 0; \text{GaAs}; E_G = 1.43 \text{ eV}$$

$$x = 1; \text{AlAs}; E_G = 2.16 \text{ eV}$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$E_G - 1.43 = \frac{2.16 - 1.43}{1 - 0}(x - 0)$$

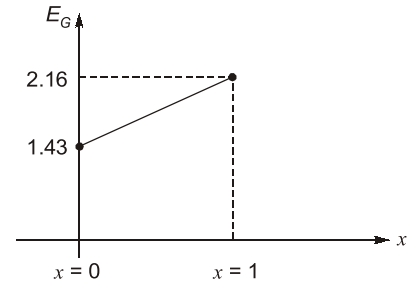
$$E_G = 1.43 + 0.73x$$

For 680 nm,

$$E_G = \frac{1240}{680} = 1.823 \text{ eV} = 1.43 + 0.73x$$

$\therefore$

$$x = 0.54$$



Level-2

1.8 (a) Show that a semiconductor has minimum conductivity at a given temperature when

$$n = n_i \sqrt{\frac{\mu_p}{\mu_n}} \text{ and } p = n_i \sqrt{\frac{\mu_n}{\mu_p}}.$$

(b) What is the expression for minimum conductivity $\sigma_{\min}$?

(c) Calculate $\sigma_{\min}$ for silicon at 300 K and compare with intrinsic conductivity.

(15 Marks)

Solution:

(a)

$$\sigma = ne\mu_n + pe\mu_p$$

$$\sigma = ne\mu_n + \frac{n_i^2}{n}e\mu_p \quad \left(\because p = \frac{n_i^2}{n} \right)$$

To find n for which σ becomes minimum,

$$\frac{d\sigma}{dn} = 0$$

$$e\mu_n - \frac{n_i^2}{n^2}e\mu_p = 0$$

$$\therefore n = n_i \sqrt{\frac{\mu_p}{\mu_n}}$$

$$\text{and } p = n_i \sqrt{\frac{\mu_n}{\mu_p}}$$

$$\text{As, } \mu_p < \mu_n \text{ and } p > n_i$$

(Minimum conductivity occurs in a very lightly doped p type semiconductor)

(b)
$$\sigma_{\min} = n_i \sqrt{\frac{\mu_p}{\mu_n}} e \mu_n + n_i \sqrt{\frac{\mu_n}{\mu_p}} e \mu_p$$

$$\sigma_{\min} = n_i e \sqrt{\mu_p \mu_n} + n_i e \sqrt{\mu_n \mu_p}$$

$$\sigma_{\min} = 2n_i e \sqrt{\mu_n \mu_p}$$

(c) For silicon at 300 K,

$$n_i = 1.5 \times 10^{10} / \text{cm}^3$$

$$\mu_n = 1300 \text{ cm}^2 / \text{Vsec}$$

$$\mu_p = 500 \text{ cm}^2 / \text{Vsec}$$

$$\begin{aligned} \sigma_{\min} &= 2 \times 1.5 \times 10^{10} \times 1.6 \times 10^{-19} \sqrt{1300 \times 500} \\ &= 3.87 \times 10^{-6} \text{ } \Omega \text{ cm}^{-1} \end{aligned}$$

$$\begin{aligned} \sigma_i &= n_i (\mu_n + \mu_p) e \\ &= (1.5 \times 10^{10}) (1800) (1.6 \times 10^{-19}) \\ &= 4.32 \times 10^{-6} \text{ } \Omega \text{ cm}^{-1} \end{aligned}$$

$$\sigma_{\min} < \sigma_i$$

(So, a very lightly doped p type SC will have less conductivity than intrinsic SC because decrease in mobility is more significant compared to increase in carrier concentration).

1.9 Obtain expression for intrinsic fermi level of a semiconductor w.r.t. conduction band edge. Estimate the shift of fermi level from the middle of band gap at 300 K for $I_n S_p$. Assume that the ratio of electron to hole mass is 0.014.

(15 Marks)

Solution:

For intrinsic semiconductor;

$$n = p$$

$$N_c e^{-(E_c - E_{F_i})/kT} = N_v e^{-(E_{F_i} - E_v)/kT}$$

$$\frac{N_c}{N_v} = e^{(-E_{F_i} + E_v + E_c - E_{F_i})/kT} = e^{(E_c + E_v - 2E_{F_i})/kT}$$

$$\ln\left(\frac{N_c}{N_v}\right) = \frac{E_c + E_v - 2E_{F_i}}{kT}$$

$$E_{F_i} = \frac{E_c + E_v}{2} - \frac{kT}{2} \ln\left(\frac{N_c}{N_v}\right) \quad [\text{Fermi level w.r.t. middle of band gap}] \quad \dots(i)$$

E_{F_i} w.r.t. E_c ;

$$E_{F_i} - E_c = \frac{E_c + E_v}{2} - \frac{kT}{2} \ln\left(\frac{N_c}{N_v}\right) - E_c$$

$$= \left(\frac{E_v - E_c}{2}\right) - \frac{kT}{2} \ln\left(\frac{N_c}{N_v}\right)$$

$$E_c - E_{F_i} = \left(\frac{E_c - E_v}{2}\right) + \frac{kT}{2} \ln\left(\frac{N_c}{N_v}\right) \quad [\text{Fermi level w.r.t. conduction band edge}]$$

$$[E_c - E_v = E_g]$$

Given;

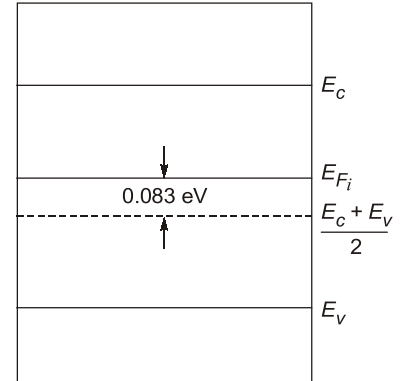
$$\frac{m_n}{m_p} = 0.014$$

$$E_{F_i} = \frac{E_c + E_v}{2} - \frac{kT}{2} \ln \left(\frac{N_c}{N_v} \right) \quad \text{from (i)}$$

$$= \frac{E_c + E_v}{2} - \frac{kT}{2} \ln \left(\frac{m_n}{m_p} \right)^{3/2}$$

$$= \frac{E_c + E_v}{2} - \frac{0.026}{2} \ln(0.014)^{3/2}$$

$$E_{F_i} = \frac{E_c + E_v}{2} + 0.083$$



So, fermi level is 0.083 eV above the centre of forbidden band.

[Note : if $m_n = m_p$, then $E_{F_i} = \frac{E_c + E_v}{2}$ i.e. fermi level will lie exactly at the centre of forbidden band]

- 1.10 A new semiconductor has $N_c = 10^{19} \text{ cm}^{-3}$, $N_v = 5 \times 10^{18} \text{ cm}^{-3}$ and $E_g = 2\text{eV}$. If it is doped with 10^{17} donors (fully ionized), calculate the electron, hole and intrinsic carrier concentrations at 627°C . Sketch the simplified band diagram showing the position of E_F .

(15 Marks)

Solution:

Given : $N_c = 10^{19} \text{ cm}^{-3}$, $N_v = 5 \times 10^{18} \text{ cm}^{-3}$, $E_g = 2\text{eV}$, $T = 627^\circ\text{C} = 900 \text{ K}$, $n = 10^{17} \text{ cm}^{-3}$

We know that,
$$n = N_c e^{-\frac{(E_c - E_F)}{kT}}$$

$$\Rightarrow E_c - E_F = -kT \ln \left(\frac{n}{N_c} \right) = -0.078 \ln \left(\frac{10^{17}}{10^{19}} \right)$$

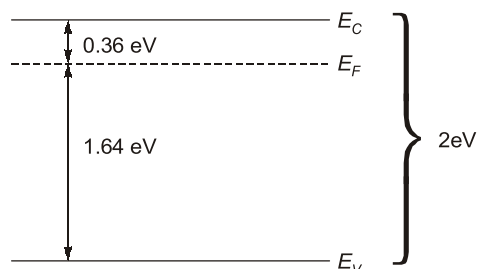
$$\therefore E_c - E_F = 0.36 \text{ eV}$$

$$\begin{aligned} \text{Now, } E_F - E_v &= [(E_c - E_v) - (E_c - E_F)] = [E_g - (E_c - E_F)] \\ &= 2\text{eV} - 0.36 \text{ eV} \\ &= 1.64 \text{ eV} \end{aligned}$$

$$\begin{aligned} p &= N_v e^{-\frac{(E_F - E_v)}{kT}} = 5 \times 10^{18} e^{-\frac{1.64}{0.078}} \\ &= 3.7 \times 10^9 \text{ cm}^{-3} \end{aligned}$$

$$\begin{aligned} n_i &= \sqrt{n \cdot p} = \sqrt{10^{17} \times 3.7 \times 10^9} \\ &= 1.9 \times 10^{13} \text{ cm}^{-3} \end{aligned}$$

[Note : $n_i = \sqrt{N_c N_v} e^{-\frac{E_g}{2kT}}$ may also be used]



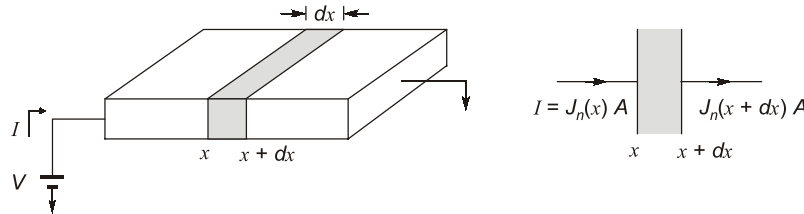
1.11 Derive one-dimensional continuity equation for (a) free electrons (b) holes in a semiconductor.

(20 Marks)

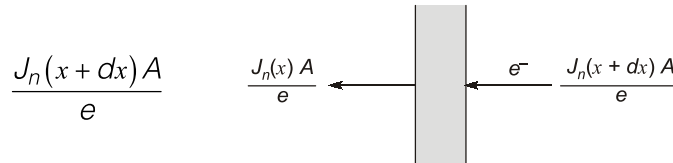
Solution:

Continuity equation is helpful in finding the net rate of change of carrier concentration when drift, diffusion, generation and recombination processes are occurring simultaneously in a semiconductor sample.

(a) • Consider n-type semiconductor sample



- Let us consider a thin slice of sample having thickness dx . Current flow in n-type sample is due to free electrons.
- The net rate of change of electron concentration in this thin slice can be calculated by considering four factors.
 - the rate at which electrons are flowing into the slice:



- the rate at which electrons are flowing out of the slice :

$$\frac{J_n(x)A}{e}$$

- the rate at which electrons are generated in the slice i.e. $G_n Adx$ (G_n : EHP/cm³/sec)

- the rate at which electrons are recombining in the slice i.e. $R_n Adx$

- the net rate of change of electron concentration in the slice is

$$\frac{\partial n}{\partial t} Adx = \frac{J_n(x+dx)A}{e} - \frac{J_n(x)A}{e} + G_n Adx - R_n Adx$$

$$\frac{\partial n}{\partial t} dx = \frac{J_n(x+dx) - J_n(x)}{e} + (G_n - R_n)dx$$

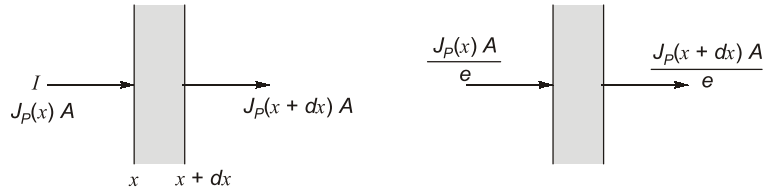
$$\frac{\partial n}{\partial t} dx = \frac{1}{e} \left[\frac{\partial J_n(x)}{\partial x} \right] dx + (G_n - R_n)dx$$

$$\left[\text{From Taylor series; } J_n(x+dx) \simeq J_n(x) + \frac{\partial J_n(x)}{\partial x} dx \right]$$

$$\therefore \frac{\partial n}{\partial t} = \frac{1}{e} \frac{\partial J_n(x)}{\partial x} + (G_n - R_n); \quad \text{Continuity equation for electrons}$$

$$\left[\text{where } J_n(x) = ne\mu_n + e\Delta_n \frac{dn}{dx} \right]$$

(b) Similarly, for p type sample;



$$\frac{\partial p}{\partial t}(Adx) = \frac{J_p(x)A}{e} - \frac{J_p(x+dx)A}{e} + (G_p - R_p)Adx$$

$$\frac{\partial p}{\partial t} dx = - \left[\frac{J_p(x+dx) - J_p(x)}{e} \right] + (G_p - R_p) dx$$

$$\frac{\partial p}{\partial t} dx = - \frac{1}{e} \frac{\partial J_p(x)}{\partial x} \cdot dx + (G_p - R_p) dx$$

$$\therefore \frac{\partial p}{\partial t} = - \frac{1}{e} \frac{\partial J_p(x)}{\partial x} + (G_p - R_p); \quad \text{Continuity equation for holes}$$

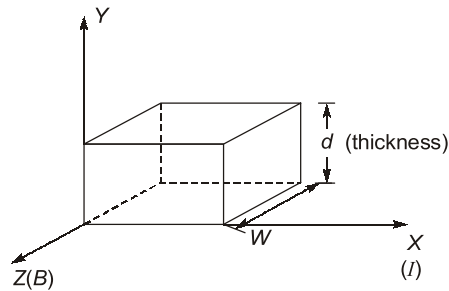
$$\left[\text{where } J_p(x) = pq\mu_p E - e\Delta_p \frac{dp}{dx} \right]$$

1.12 Explain Hall effect. An n -type Germanium sample is 2 mm wide and 0.2 mm thick. A current of 10 mA is passed through the sample in x -direction and a magnetic field of 0.1 Wb/m² is applied perpendicular to current flow in z -direction. The developed Hall voltage is -1.0 mV. Calculate the Hall coefficient and density of electrons.

(20 Marks)

Solution:

- Hall effect occurs in metals and semiconductors. It refers to development of electric field in a current carrying sample when it is placed in a perpendicular magnetic field.



- Consider a sample in which current is due to electrons. If current flows in positive x -direction, the electron velocity will be in negative x -direction.
- When perpendicular magnetic field in z -direction is applied then, Lorentz force will act on electrons

$$\text{Lorentz force} = q(\vec{v} \times \vec{B})$$

- Lorentz force on electrons is in downward direction. Hence electrons bend down and lower surface gets negatively charged. Relatively upper surface of sample becomes positive. Hence, an electric field develops in the sample in downward direction. It is called Hall electric field.
- When Hall field becomes sufficiently strong, it will exert an additional force on electrons in upward direction.
- When force due to Hall field becomes equal to Lorentz force, electrons are said to be in equilibrium.

In equilibrium,

Force due to E_H = Force due to B

$$eE_H = eBV$$

$$E_H = BV;$$

Hall electric field

$$\frac{V_H}{d} = B \frac{J}{\rho_v}$$

$$(\because J = \rho_v V)$$

$$\frac{V_H}{d} = \frac{B}{\rho_v} \cdot \frac{I}{A} = \frac{B}{\rho_v} \cdot \frac{I}{W \cdot d}$$

$$V_H = \frac{BIR_H}{W} \quad \left[\text{where } R_H = \frac{1}{\rho_v} \text{ is Hall coefficient} \right]$$

n-type Ge sample

$W = 2 \text{ mm}$, $d = 0.2 \text{ mm}$, $I = 10 \text{ mA}$, $B = 0.1 \text{ Wb/m}^2$, $V_H = -1 \text{ mV}$, $R_H = ?$ $n = ?$

$$V_H = \frac{BIR_H}{W}$$

$$1 \times 10^{-3} = \frac{0.1 \times 10 \times 10^{-3} \times R_H}{2 \times 10^{-3}}$$

$$R_H = \frac{10^{-3}}{0.5} = 2 \times 10^{-3} \text{ m}^3/\text{C}$$

$$R_H = \frac{1}{ne}$$

$$\Rightarrow n = \frac{1}{R_H e} = \frac{1}{2 \times 10^{-3} \times 1.6 \times 10^{-19}} = 3.125 \times 10^{21} / \text{m}^3$$

- 1.13 Hall measurements are made on a p-type semiconductors bar $500 \mu\text{m}$ wide and $20 \mu\text{m}$ thick. The Hall contacts A and B are displaced $2 \mu\text{m}$ with respect to each other in the direction of current flow of 3 mA . The voltage between A and B with a magnetic field of 10 kg ($1 \text{ kg} = 10^{-5} \text{ Wb/cm}^2$) pointing out of the plane of sample is 3.2 mV . When the magnetic field direction is reversed, the voltage changes to -2.8 mV . What is the hole concentration and mobility?

(20 Marks)

Solution:

Displacement of the probes by an amount δ given a small IR drop $V\delta$ in addition to V_H . The hall voltage reverses when the magnetic field is reversed. However, $V\delta$ does not depend on the direction of magnetic field.

For positive magnetic field $V_{AB}^+ = V_H + V\delta$

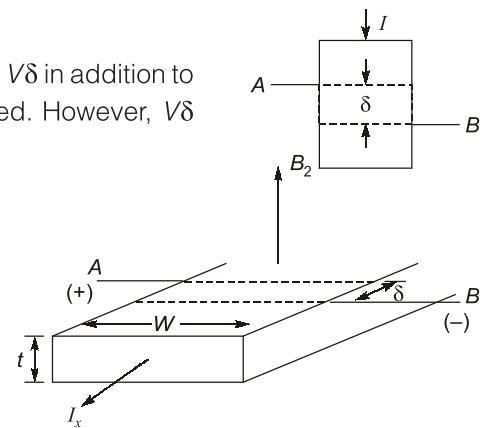
For negative magnetic field $V_{AB}^- = -V_H + V\delta$

$$\therefore V_{AB}^+ - V_{AB}^- = 2V_H$$

$$\Rightarrow V_H = \frac{V_{AB}^+ - V_{AB}^-}{2}$$

Given : $W = 500 \mu\text{m}$, $t = 20 \mu\text{m}$, $\delta = 2 \mu\text{m}$, $I_x = 3 \text{ mA}$, $B_z = 10^{-4}$

Wb/cm^2 , $V_{AB}^+ = 3.2 \text{ mV}$, $V_{AB}^- = -2.8 \text{ mV}$



$$V_H = \frac{V_{AB}^+ - V_{AB}^-}{2} = \frac{3.2 - (-2.8)}{2} = 3 \text{ mV}$$

$$\therefore \text{Ohmic drop} = (3.2 - 3) \text{ mV} = 0.2 \text{ mV}$$

$$R = \frac{0.2 \text{ mV} \left(\frac{\text{ohmic drop}}{\text{current}} \right)}{3 \text{ mA}}$$

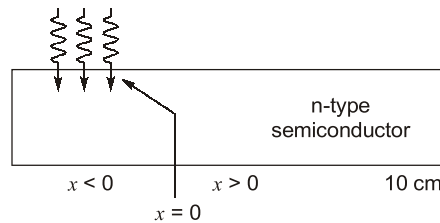
$$\rho = \frac{RA}{l} = \frac{0.2 \times 10^{-3} \times 5 \times 10^{-2} \times 2 \times 10^{-3}}{3 \times 10^{-3} \times 2 \times 10^{-4}} = 0.033 \Omega \text{ cm}$$

$$\text{Now, } V_H = \frac{B_z I_x R_H}{t} = \frac{B_z I_x}{pe t} \quad \left(\because R_H = \frac{1}{pe} \right)$$

$$\therefore \rho = \frac{B_z I_x}{V_H e t} = \frac{10^{-4} \times 3 \times 10^{-3}}{3 \times 10^{-3} \times 1.6 \times 10^{-19} \times 2 \times 10^{-3}} = 3.125 \times 10^{17} \text{ cm}^{-3}$$

$$\text{Now, } \mu = \frac{\sigma}{pe} = \frac{1}{\rho(pe)} = \frac{1}{0.033 \times 3.125 \times 10^{17} \times 1.6 \times 10^{-19}} = 600 \text{ cm}^2/\text{Vsec}$$

- 1.14 An n-type silicon bar is doped uniformly by phosphorus atoms to a concentration of $4.5 \times 10^{15} \text{ cm}^3$. The bar has a cross-section of 1 mm^2 and a length of 10 cm . It is illuminated uniformly for region $x < 0$ as shown in the figure. Assume optical generation rate 10^{21} electron-hole pairs per cm^3/sec . The hole lifetime and electron lifetime are equal and are $1 \mu\text{sec}$. Find the hole and electron diffusion current at $x = 34.6 \mu\text{m}$.



(20 Marks)

Solution:

$$n = N_D = 4.5 \times 10^{15} / \text{cm}^3$$

$$p = \frac{n_i^2}{N_D} = \frac{2.25 \times 10^{20}}{4.5 \times 10^{15}} = 5 \times 10^4 / \text{cm}^3$$

$$\tau_p = \tau_n = 10^{-6} \text{ sec}$$

$$D_p = V_T \mu_p = 0.026 \times 500 = 13 \text{ cm}^2/\text{sec}$$

$$L_p = \sqrt{D_p \tau_p} = \sqrt{13 \times 10^{-6}} = 3.6 \times 10^{-3} \text{ cm}$$

$$D_n = V_T \mu_n = 0.026 \times 1300 = 33.8 \text{ cm}^2/\text{sec}$$

$$L_n = \sqrt{D_n \tau_n} = 5.81 \times 10^{-3} \text{ cm}$$

$$\Delta n|_{x=0} = \Delta p|_{x=0} = G_L \tau_p = 10^{21} \times 10^{-6} = 10^{15} / \text{cm}^3$$

Excess hole concentration at any distance x :

$$\Delta p(x) = 10^{15} e^{-\frac{x}{L_p}} \quad \text{for } x > 0$$

$$\Delta n(x) = 10^{15} e^{-\frac{x}{L_n}} \quad \text{for } x > 0$$

Total hole concentration : $p(x) = p + \Delta p(x) = p + 10^{15} e^{-\frac{x}{L_p}}$

Hole diffusion current, $I_p(x) = -AeD_p \frac{dp}{dx}$

$$= -10^{-2} \times 1.6 \times 10^{-19} \times 13 \times 10^{15} \left(\frac{1}{L_p} \right) e^{-\frac{x}{L_p}}$$

$$= \frac{10^{-2} \times 1.6 \times 10^{-19} \times 13 \times 10^{15}}{3.6 \times 10^{-3}} e^{-\frac{x}{(3.6 \times 10^{-3})}}$$

$$= 5.77 \times 10^{-3} \times e^{-\frac{34.6 \times 10^{-4}}{3.6 \times 10^{-3}}} = 2.2 \text{ mA}$$

Electron diffusion current, $I_n(x) = Ae\Delta n \frac{dn}{dx}$

$$= 10^{-2} \times 1.6 \times 10^{-19} \times 33.8 \times 10^{15} \times \left(-\frac{1}{L_n} \right) e^{-\frac{x}{L_n}}$$

$$= \frac{10^{-2} \times 1.6 \times 10^{-19} \times 33.8 \times 10^{15} \times e^{-\frac{34.6 \times 10^{-4}}{5.81 \times 10^{-3}}}}{5.81 \times 10^{-3}} = -5.13 \text{ mA}$$

2. PN Junction Diode

Level-1

2.1 Derive I-V relation for a diode under ideal conditions.

(10 Marks)

Solution:

Consider a P-N junction operated in forward bias. At the junction net current is sum of electron and hole diffusion currents.

$$I = I_{pn}(0) + I_{np}(0) \quad \dots(i)$$

The net hole concentration on n-side is the sum of injected hole concentration and thermally generated hole concentration.

