

ESE 2027

UPSC ENGINEERING SERVICES EXAMINATION

Main Examination



Electrical Engineering

Topicwise *Conventional* Solved Papers

Paper-I

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ESE-2027 : Main Examination

Electrical Engineering : Paper-I | Conventional Solved Questions : (2002-2026)

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B. Singh (Ex. IES)

Director's Message

In past few years ESE Main exam has evolved as an examination designed to evaluate a candidate's subject knowledge. Studying engineering is one aspect but studying to crack prestigious ESE exam requires altogether different strategy, crystal clear concepts and rigorous practice of previous years' questions. ESE mains being conventional exam has subjective nature of questions, where an aspirant has to write elaborately - step by step with proper and well labeled diagrams and figures. This characteristic of the main exam gave me the aim and purpose to write this book. This book is an effort to cater all the difficulties being faced by students during their preparation right from conceptual clarity to answer writing approach.

MADE EASY Team has put sincere efforts in solving and preparing accurate and detailed explanation for all the previous years' questions in a coherent manner. Due emphasis is made to illustrate the ideal method and procedure of writing subjective answers. All the previous years' questions are segregated subject wise and further they have been categorised topic-wise for easy learning and helping aspirants to solve all previous years' questions of particular area at one place. This feature of the book will also help aspirants to develop understanding of important and frequently asked areas in the exam.

I would like to acknowledge the efforts of entire MADE EASY team who worked hard to solve previous years' questions with accuracy. I hope this book will stand upto the expectations of aspirants and my desire to serve the student community by providing best study material will get accomplished.

B. Singh (Ex. IES)
CMD, MADE EASY Group

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Electrical Materials

Revised Syllabus of ESE: *Electrical engineering materials, crystal structures and defects, ceramic materials, insulating materials, magnetic materials-basics, properties and applications, ferrites, ferro-magnetic materials and components, basics of solid state physics, conductors, photo-conductivity, basics of nano materials and superconductors.*

1. Introduction to Engineering Materials

1.1 What is the basis for classifying a material as a conductor, semi-conductor or a dielectric? What is the conductivity of a perfect dielectric?

[12 marks : 2002]

Solution:

Classification of insulators, metals and semiconductors are mainly by its conductivity and its temperature coefficient of resistivity.

Conductor:

In which conduction band and valence band are overlapping and there is no band gap, between them ($\Delta E_g = 0$). Due to this large number of electrons available for electrical conduction and therefore its resistivity is low and temperature coefficient of resistivity is positive ($\rho = 10^{-2} \Omega\text{m}$ to $10^{-8} \Omega\text{m}$).

Insulator:

In which energy gap is large ($E_g > 3\text{eV}$). So in conduction band there are no electrons and so no electrical conduction is possible. Here energy gap is so large that electrons cannot be easily excited from the valence band to conduction band by any external energy.

$$\rho = 10^8 \Omega\text{-m}$$

Temperature coefficient of resistivity is negative

Semiconductor:

In which a finite but small band gap ($E_g < 3\text{eV}$) exists. Due to this small band gap some electrons can be thermally excited to "conduction band". These thermally excited electrons can move in conduction band and can conduct current, their resistivity is in medium range. Its $\rho = 10^5 - 10^{10} \Omega\text{m}$. temperature coefficient of resistance is negative. Conductivity of a perfect dielectric is zero.

1.2 Write the notable shortcoming of the free electron model. How it is overcome by band-theory of solids? Sketch the energy bands of diamond. With the help of energy band diagram, explain an insulator.

[12 marks : 2003]

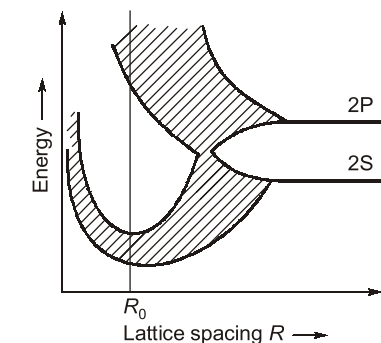
Solution:

In free electron theory developed by Arnold Sommerfeld we assume that the electron to be completely free. But it fails mainly because most electrons are not completely free. They interact with

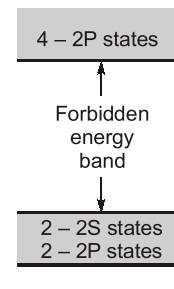
- (i) lattice (ii) with each other (iii) impurities

Drude incorporated into his model in the form of a hard sphere interaction but completely neglected the other interactions. Sommerfeld made some corrections to this purely classical model by ensuring that the electron gas obey Fermi statistics. This faced some problems (like the overestimate of the electronic heat capacity but still didn't cover for the missing interaction in the model). Due to its limitations, the free electron model was unable to account for things like magnetoresistance, thermal dependence of conductivity, optical properties and the host of many body phenomenon that result from electron-electron interaction being of importance. In band theory of e^- s, a solid is assumed to contain many bands in which electrons in it are packed. The most important are valence band and conduction band. The energy of electrons in these bands will be different. The difference in valence band and conduction determines whether solid is a conductor, semiconductor, or insulator.

So it clearly explains the dependency of conductivity on temperature which can't be explained by free e^- theory. When temperature increases, the energy of e^- s increases, so e^- s started to jump from valence band to conduction so conduction of e^- s changes with temperature in CB so as conductivity changes with temperature. It also explains many other properties like optical, magneto resistance etc. easily so shortcomings of free e^- theory is overcome by band theory in which we assume that there is an interaction between e^- s only when energy state of different e^- s is different. The interaction of e^- with lattice and impurities are taken into account. Diamond is the crystalline form of carbon and is of interest because it lies in the same period and has the same type of bonding as silicon and germanium. The shape of the energy-band structure of diamond as a function of atomic spacing is shown in Fig. (a), it is shown that there is a crossover point at which two of the 2 P states form a band with the two 2S state, thus making available a band with four available states. The remaining four of the total of six 2 P states off and form a higher band of their own. At normal atomic spacing, the forbidden gap between the higher and lower bands is wide. Since there is a total of four electrons in the 2 S and 2 P states of carbon, all these electrons are used up in filling the lower band. The upper band is left empty, and hence diamond is a very good insulator. Fig. (b) shows the stable energy level of diamond.



(a) The energy levels in the tetrahedral of diamond as a function of lattice spacing.



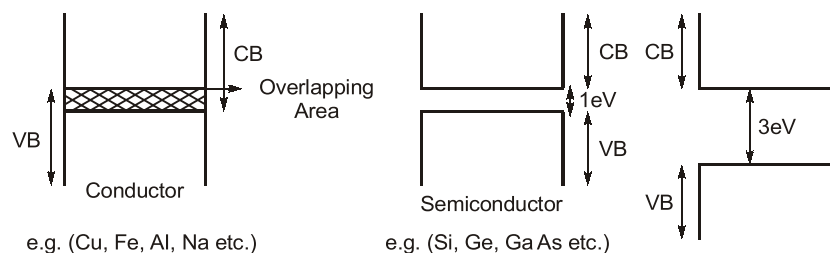
(b) The stable energy level of the diamond crystal.

1.3 Distinguish between metals, insulators and semiconductors.

[12 marks : 2007]

Solution:

According to Energy Band Theory:



Classification of insulators, metals and semiconductors mainly by its conductivity and its temperature coefficient of resistivity.

Conductor:

In which conduction band and valence band are overlapping and there is no band gap, between them it $\Delta E_g = 0$. Due to this large number of electrons available for electrical conduction and therefore its resistivity is low and temperature coefficient of resistivity is positive ($\rho = 10^{-2}$ to $10^{-8} \Omega\text{-m}$).

Insulator:

In which energy gap is large ($E_g > 3 \text{ eV}$). So in conduction band there are no electrons and so no electrical conduction is possible. Here energy gap is so large that electrons cannot be easily excited from the valence band to conduction band by any external energy.

$$\rho = 10^8 \Omega\text{-m}$$

Temperature coefficient of resistivity is negative.

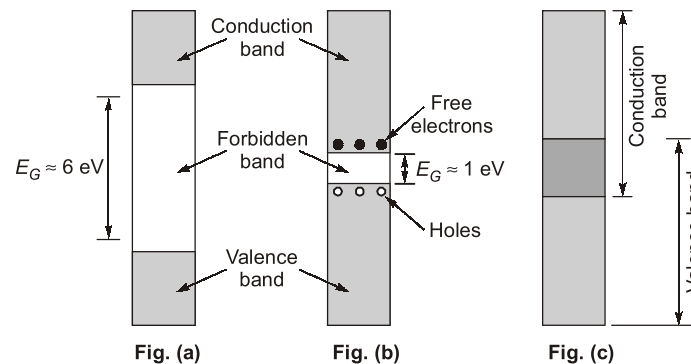
Semi-conductor:

In which a finite but small band gap ($E_g < 3 \text{ eV}$) exists. Due to this small band gap some electrons can be thermally excited to "conduction band". These thermally excited electrons can move in conduction band and can conduct current, their resistivity is in medium range. Its $\rho = (10^5 \text{ to } 10^{10}) \Omega\text{-m}$ temperature coefficient of resistance is negative. Conductivity of a perfect dielectric is zero.

1.4 Explain with the diagram band structure of metals, insulators and semiconductors.

[10 marks : 2013]

Solution:



Insulators:

- An insulating material has an energy band diagram as shown in figure (a).
- It has a very wide forbidden-energy gap ($\approx 6 \text{ eV}$) separating the filled valence region from the vacant conduction band. Because of this, it is practically impossible for an electron in the valence band to jump the gap, reach the conduction band.
- At room temperature, an insulator does not conduct. However it may conduct if its temperature is very high or if a high voltage is applied across it. This is termed as the **breakdown of the insulator**.
- **Example:** diamond.

Semiconductors:

- A semiconductor has an energy-band gap as shown in figure (b).
- At 0°K semiconductor materials have the same structure as insulators except the difference in the size of the band gap E_g , which is much smaller in semiconductors ($E_g \approx 1 \text{ eV}$) than in insulators.
- The relatively small band gaps of semiconductors allow for excitation of electrons from the lower (valence) band to the upper (conduction) band by reasonable amount of thermal or optical energy.
- The difference between semiconductors and insulators is that the conductivity of semiconductors can increase greatly by thermal or optical energy.
- **Example:** Ge and Si.

Metals:

- There is no forbidden energy gap between the valence and conduction bands. The two bands actually overlap as shown in figure (c).
- Without supplying any additional energy such as heat or light, a metal already contains a large number of free electrons and that is why it works as a good conductor.
- **Example:** Al, Cu etc.

1.5 State three effects of moisture on insulation and explain any two methods adopted to protect them against moisture along with salient features.

[12 marks : 2017]

Solution:**The effect of moisture on insulation:**

- Change in electrical properties:** Moisture absorbed by the insulation causes a decrease in the volume resistivity, and especially surface resistivity, an increase in the dissipation factor and a certain increase in dielectric constant, a reduction in the dielectric strength due to change in field distribution within the insulating material. Under high humidity and electric tension current, conducting bridge may appear across the surface of the insulating material. In some cases the thin films formed by the moisture on insulating material dries up when the equipment is working. Such places get a carbonized spot. Such carbonized spot may join together with time and build up a conducting bridge, thus a short-circuit may result.
- Physical and mechanical changes:** Due to high humidity for long time some insulating materials like plastics polymers and material filled with cellulose fillers swell. The swelling of inner and outer layers of insulating material are unequal, so small cracks appear on the surface layer, through which moisture easily penetrates into the material. Mechanical strength of the insulating material looses with moisture.
- Chemical changes:** High humidity often causes hydrolysis. For example under the effect of moisture, linseed base insulating vanishes soften and even change into liquid. This evolves organic acids which deteriorate other organic materials and bring about intensive corrosion of metals. High humidity favours the growth of fungi in some insulating materials. The products of life activity formed by fungi degrade organic insulating materials.

Protection of insulation against moisture:

- Impregnation of winding:** The windings of all low voltages pieces of equipment are impregnated with baking varnishes and some time with compounds. This treatment solidifies the windings, increases their thermal conductivity, improves their electrical and mechanical strength and heat resistance. Impregnating varnishes and compounds raise the moisture resistance of windings.
- Rendering insulation hydrophobic:** Sometimes the insulating materials assemblies are rendered hydrophobic (or waterproofed) so as to protect them against moisture. This treatment is particularly effective for polymer containing hydrolysis and for cellulose-base insulating materials. It is a good practice to render some inorganic materials hydrophobic so as to decrease their wettability and to raise the surface resistivity of these materials. In addition to old widely used techniques employing asphalts, bitumens, waxes, waterproofing by means of some hydrophobic silicon compositions free of hydroxyls and carboxyls is finding ever increasing favour. Paper, cotton fabrics are rendered hydrophobic by dipping them in the solution of methyl butoxidiamine silane in carbon tetrachloride or methyl triethoxysilane in absolute alcohol.

1.6 Show that the surface area to volume ratio increases with the reduction in radius of nanoparticle. Plot a graph of surface to volume ratio as a function of radius.

[8 marks : 2017]

Solution:**Surface to volume ratio of nanoparticle relation:**

Surface to volume ratio in nanoparticles have a significant effect on the nanoparticles properties firstly, nanoparticles have a relative large surface area when compared to the same volume of the material.

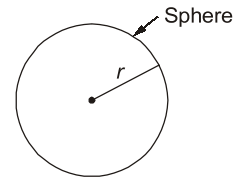
Let us consider a sphere of radius ' r '.

The surface area of the sphere, $S = 4\pi r^2$

and the volume of sphere, $V = \frac{4}{3}\pi r^3$

Therefore the surface area to volume ratio,

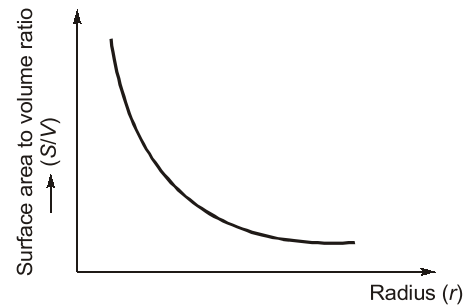
$$\frac{S}{V} = \frac{4\pi r^2}{\left(\frac{4}{3}\pi r^3\right)} = \frac{3}{r} \Rightarrow \frac{S}{V} \propto \frac{1}{r}$$



It means that the surface area to volume ratio increases with the decreases in radius of the sphere and vice-versa. It can also be concluded that when given volume is divided into smaller piece, the surface area increases. Therefore as particle size decreases, a greater portion of the atoms are found at the surface compared to those inside.

For example, a particle of size 3 nm has 50% of its atom on its surface, at 10 nm 20% of its atoms and at 30 nm has 5% of its atoms on its surface.

Therefore nanoparticles have a much greater surface area per unit volume compared with the large particles. It leads to nanoparticles more chemically reactive. As growth and catalytic chemical reaction occur at surfaces, therefore a given mass of nanomaterial will be much more reactive than the same mass of nano material made up of large particles. It is also found that material which are insert in their bulk form are reactive when produced in their nanoscale form. It can improve their properties. Surface area to volume ratio vs radius plots.



$$\frac{S}{V} = \frac{3}{r}; \quad \frac{S}{V} \propto \frac{1}{r}$$

- 1.7** Determine the diameter of the lead wire to pass maximum super current of 0.0021 mA at 5.0 K. The critical temperature and critical magnetic field for lead are respectively 7.18 K and 6.5×10^{-4} A/m. [8 marks : 2017]

Solution:

Given that:

Maximum supercurrent, $I_m = 0.0021$ mA at 5.0 K

The critical temperature = 7.18 K

The critical magnetic field = 6.5×10^{-4} A/m

We know that,

$$H_C = H_0 \left[1 - \left(\frac{T}{T_C} \right)^2 \right]$$

$$\begin{aligned} H_{C(5K)} &= H_{0(7.18K)} \left[1 - \left(\frac{5}{7.18} \right)^2 \right] \\ &= 6.5 \times 10^{-4} [1 - 0.48494] = 6.5 \times 10^{-4} (0.51506) \\ &= 3.3479 \times 10^{-4} \text{ A/m} \end{aligned}$$

Now, maximum current, $I_m = 2\pi r \cdot H_C$
 $0.0021 \times 10^{-3} = 2\pi r \times 3.3479 \times 10^{-4}$

Radius of lead wire, $r = \frac{0.0021 \times 10^{-3}}{2\pi \times 3.3479 \times 10^{-4}} = 9.9831 \times 10^{-4} \text{ m}$
 $= 0.99831 \times 10^{-3} \text{ m} = 0.99831 \text{ mm}$

So, diameter of the lead wire,
 $d = 2r = 2 \times 0.99831 = 1.99662 \text{ mm} \simeq 2.0 \text{ mm}$

1.8 Mention the type of Bravais space lattice, relationship of crystal axial lengths (x, y, z) and relationship of interaxial angles (a, b, g) in the following order of the crystal system.

- (i) Triclinic (ii) Monoclinic
 (iii) Orthorhombic and (iv) Trigonal

[12 marks : 2018]

Solution:

Crystal System	Axial Relationships	Interaxial Angles
Cubic	$a = b = c$	$\alpha = \beta = \gamma = 90^\circ$
Hexagonal	$a = b \neq c$	$\alpha = \beta = 90^\circ, \gamma = 120^\circ$
Tetragonal	$a = b \neq c$	$\alpha = \beta = \gamma = 90^\circ$
Rhombohedral (Trigonal)	$a = b = c$	$\alpha = \beta = \gamma \neq 90^\circ$
Orthorhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90^\circ$
Monoclinic	$a \neq b \neq c$	$\alpha = \gamma = 90^\circ \neq \beta$
Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90^\circ$

1.9 Compute the packing efficiency and density of diamond. Consider diamond as diamond cubic (DC) structure and lattice parameter, $a = 3.57 \text{ \AA}$ and mass of atom $m = 1.66 \times 10^{-27} \text{ kg}$.

[12 marks : 2022]

Solution:

Packing Efficiency of Diamond :

Atoms present at CCP = 4

Atoms present at alternate $T_o V_o = 4$

So, total number of atoms in diamond,

$$Z = 8$$

So,
$$\text{pf} = \frac{Z \times \frac{4}{3} \times \pi r^3}{a^3}$$

...(i)

Now to find relation between a and r .

We know atoms touch corner with TV.

$$\text{So, } \frac{\sqrt{3}a}{4} = 2r$$

$$a = \frac{8r}{\sqrt{3}} \quad \dots(ii)$$

Using (ii) in (i), we get,

$$\text{pf} = \frac{\frac{4}{3} \times \pi \times 8 \times r^3}{\left(\frac{8r}{\sqrt{3}}\right)^3} = 0.34 \text{ or } 34\%$$

To find density :

$$\rho = \frac{Z \times m}{a^3}$$

$$\text{Using the values given, } m = 1.66 \times 10^{-27} \text{ kg}$$

$$a = 3.57 \times 10^{-10} \text{ m}$$

$$\text{We get, } \rho = \frac{8 \times 1.66 \times 10^{-27}}{(3.57 \times 10^{-10})^3} = 0.291 \times 10^3$$

$$\rho = 291 \text{ kg/m}^3$$

- 1.10** Zinc has HCP structure. The height of the unit cell is 0.494 Å. The nearest neighbour distance is 0.27 nm. The atomic weight of zinc is 65.37. Evaluate,
- The volume of the unit cell.
 - The density of zinc
- Take $N_A = 6.0238 \times 10^{26}$ Molecules/Kg Mole.

[12 marks : 2022]

Solution:

Given, Nearest neighbour distance,

$$a = 2r = 0.27 \text{ nm} = 0.27 \times 10^{-9} \text{ m}$$

Atomic weight of $Z_n = 65.37$

The height of the unit cell, $h = 0.494 \text{ Å}$

(i) Volume of unit cell,

The volume of the HCP unit cell is given by

$$V = \frac{3\sqrt{3}a^2h}{2} = \frac{3\sqrt{3}a^2h}{2}$$

Substituting the values we get

$$V = \frac{3\sqrt{3}(0.27 \times 10^{-9})^2 \times 0.494 \times 10^{-10}}{2} = 9.35 \times 10^{-30} \text{ m}^3$$

(ii) The density of zinc,

Effective number of atom's in unit cell of HCP,

$$n = 6$$

Using the relation, $\rho = \frac{nm}{N_0 \times V}$ we have,

[Where $n = 6$, $m = 65.37$ (given) $N_A = 6.0238 \times 10^{26}$]

$$\rho = \frac{6 \times 65.37}{6.0238 \times 10^{26} \times 9.35 \times 10^{-30}} \approx 69638 \text{ kg/m}^3$$

- 1.11** Calculate the value of contact potential and the total width of the depletion region under unbiased condition of an abrupt silicon P-N junction at 300° K. The intrinsic carrier concentration of silicon is 1.7×10^{16} atoms/m³ and its relative permittivity is 11.8. The junction has acceptor density $N_A = 10^{22}$ atoms/m³ on P-side and donor density of $N_D = 10^{20}$ atoms/m³ on n-side.

Derive the equation used for calculation.

Boltzmann's constant = 1.381×10^{-23} J/°K

Electron charge = 1.602×10^{-19} C

[20 marks : 2022]

Solution:

The contact potential of the P-N junction is

$$V_{\text{contact}} = \frac{KT}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

$$V_{\text{contact}} = \frac{1.381 \times 10^{-23} \times 300}{1.602 \times 10^{-19}} \ln \left(\frac{10^{22} \times 10^{20}}{(1.7 \times 10^{16})^2} \right)$$

$$V_{\text{contact}} = \frac{1.381 \times 10^{-23} \times 300}{1.602 \times 10^{-19}} \ln \left(\frac{10^{42}}{(1.7)^2 10^{32}} \right)$$

$$= 258.6 \times 10^{-4} \ln \left(\frac{10^{10}}{(1.7)^2} \right)$$

$$= 21.96 \times 258.6 \times 10^{-4} = 0.568 \text{ V}$$

Width of depletion region, $W = \sqrt{\frac{2 \epsilon_r \epsilon_0 V_{\text{contact}}}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right)}$

$$= \sqrt{\frac{2 \times 11.8 \times 8.85 \times 10^{-12} \times 0.568}{1.6 \times 10^{-19}} \left(\frac{1}{10^{22}} + \frac{1}{10^{20}} \right)}$$

$$W = 2.735 \times 10^{-6} \text{ m}$$

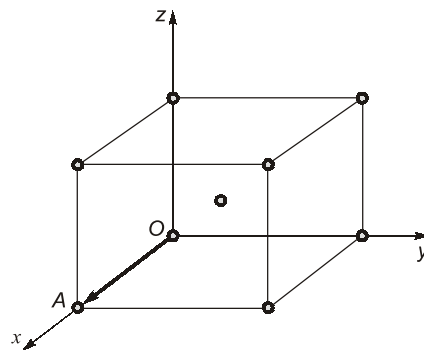
- 1.12** Evaluate the linear density in atoms per mm in the following directions in BCC iron, which has lattice constant of 2.89 Å :

(i) [1 0 0]; (ii) [1 1 0]; (iii) [1 1 1]

[12 marks : 2023]

Solution:

(i) BCC Unit Cell :



Vector OA is along [1 0 0]

Length of vector OA = Edge length of cube
= $a = 2.89 \text{ Å}$

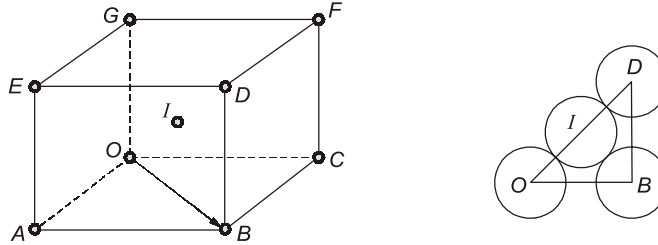
No. of atoms centred on $OA = \frac{1}{2} + \frac{1}{2} = 1$ atom

So, linear density along OA

i.e., $LD_{[100]} = \frac{1 \text{ atom}}{\text{Length of } OA} = \frac{1 \text{ atom}}{2.89 \times 10^{-7} \text{ mm}}$

$\Rightarrow LD_{[100]} = 3.46 \times 10^6 \text{ atoms/mm}$

(ii) Vector OB is along $[1\ 1\ 0]$



$\therefore OB = a\sqrt{2}$

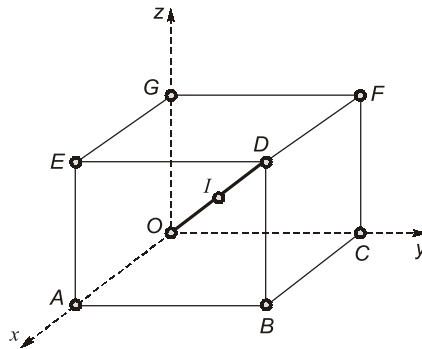
$LD_{[110]} = \frac{\text{No. of atoms centred on } [110] \text{ direction}}{\text{Length of } [110] \text{ direction vector}}$

$= \frac{\left(\frac{1}{2} + \frac{1}{2}\right) \text{ atom}}{OB} = \frac{1 \text{ atom}}{a\sqrt{2}}$

$\Rightarrow LD_{[110]} = \frac{1 \text{ atom}}{\sqrt{2} \times 2.89 \times 10^{-7} \text{ mm}} = 2.446 \times 10^6 \text{ atoms/mm}$

$\Rightarrow LD_{[110]} = 2.446 \times 10^6 \text{ atoms/mm}$

(iii) Vector OD is $[1\ 1\ 1]$ along.



$OD = \sqrt{3}a$

Number of atoms entered on $[1\ 1\ 1]$

$= \frac{1}{2} + 1 + \frac{1}{2} = 2$

$\therefore LD_{[111]} = \frac{2 \text{ atoms}}{\sqrt{3}a} = \frac{2 \text{ atoms}}{\sqrt{3} \times 2.89 \times 10^{-7} \text{ mm}} = 3.99 \times 10^6 \text{ atoms/mm}$

$\Rightarrow LD_{[111]} \simeq 4 \times 10^6 \text{ atoms/mm}$

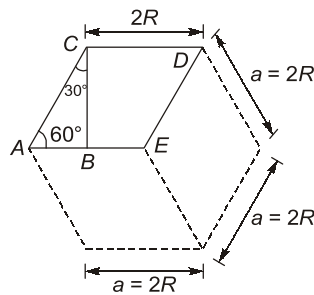
- 1.13** (i) Determine the volume of an HCP unit cell in terms of its a and c lattice parameters.
 (ii) Copper has an atomic radius of 0.13 nm, an FCC crystal structure and an atomic weight of 63.5 g/mol. Evaluate its theoretical density and compare the answer with its measured density. (Take Avogadro number, $N_A = 6.022 \times 10^{23}$ atoms/mol).

[10+10 = 20 marks : 2023]

Solution:

- (i) HCP unit cell is shown in adjacent figure :

$$\frac{c}{a} = 1.633 \text{ for HCP}$$

Volume (V_c) of HCP unit cell = Base area $\times$ Cell height (c) $\Rightarrow$ To calculate base area consider following figure :

$$BC = 2R \cos 30^\circ = \frac{2R\sqrt{3}}{2}$$

$$\Rightarrow BC = \sqrt{3}R$$

$$a = 2R \Rightarrow R = \frac{a}{2}$$

$$\text{The area of } ACDE = CD \times BC = 2R \times \sqrt{3}R$$

The base area is just three times the area of the parallelepiped $ACDE$ shown above.

$$\therefore \text{Area of base} = 3 \times \text{Area of } ACDE = 3 \times 2R \times \sqrt{3}R$$

$$= 6\sqrt{3}R^2 = 6\sqrt{3} \times \left(\frac{a}{2}\right)^2 = \frac{6\sqrt{3}}{4}a^2$$

Hence, volume of unit cell

$$V_c = \text{Area} \times c = \frac{6\sqrt{3}}{4}a^2 \times c = 2.598a^2c$$

- (ii) Theoretical density,

$$\rho = \frac{nA_{\text{Cu}}}{V_c N_A}$$

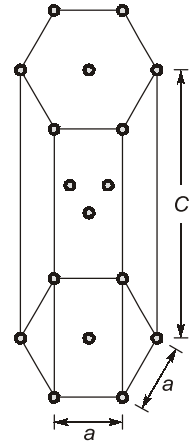
where,

 n = Number of atoms per unit cell A_{Cu} = Atomic weight of Cu V_c = Volume of unit cell

$$V_c = a^3 = (2\sqrt{2}R)^3$$

$$\Rightarrow V_c = 16\sqrt{2}R^3$$

$$\therefore \rho = \frac{4 \times 63.5}{16\sqrt{2}(0.13 \times 10^{-7})^3 \times 6.023 \times 10^{23}} \text{ g/cm}^3$$



$$\rho = 8.48 \text{ g/cm}^3 \text{ (Theoretical density of Cu)}$$

The literature value (measured value) for the density of Cu is 8.94 g/cm^3 , which is in very close agreement with above calculated value.

1.14 Define line defects in materials. Explain different types of line defects and compare them. Also, explain their cause of creation.

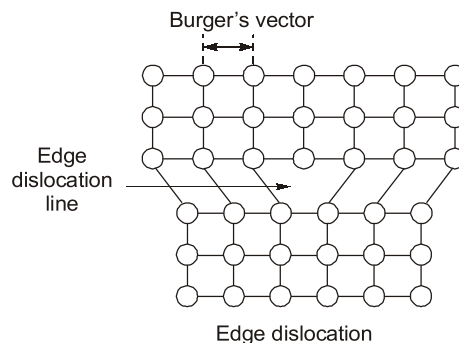
[12 marks : 2023]

Solution:

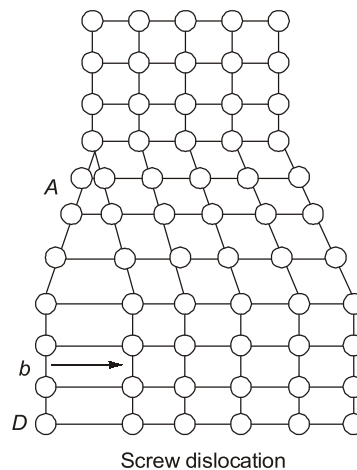
Line defect is the defect confined to more number of atoms in a lattice.

Linear defect: A dislocation is a linear or one dimensional defect around which some of the atoms are misaligned. There are following types of linear dislocation as given below:

- 1. Edge dislocation:** In an edge dislocation, an extra portion of a plane of atoms, or half-plane appears and the edge of which terminates within the crystal. It is a linear defect that centers around the line that is defined along the end of the extra half plane of atoms. This is sometimes termed as dislocation line, which for the edge dislocation as shown in figure, is perpendicular to the plane of the page. Within the region around the dislocation line there is some localized lattice distortion. The atoms above the dislocation line are squeezed together and those below are pulled apart. This is reflected in the slight curvature for the vertical planes of atoms as they bend around this extra half plane. The magnitude of this dislocation decreases with distance away from the dislocation line. An edge dislocation may also be formed by an extra half plane of atoms that is included in the bottom portion of the crystal.



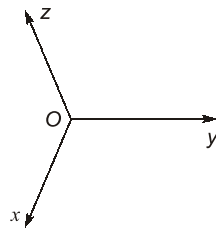
- 2. Screw dislocation:** Screw dislocation is formed by shear stress that is applied to produce the distortion. The upper front region of the crystal is shifted one atomic distance to the right relative to the bottom portion. The atomic distortion associated with a screw dislocation is also linear and along a dislocation line. The screw dislocation derives its name from the spiral or helical path or ramp that is traced around the dislocation line by the atomic planes of atoms.



3. **Mixed dislocation:** Most dislocations found in crystalline materials are probably neither pure edge nor pure screw, but exhibit components of both types, these are termed as mixed dislocations.

The nature of a dislocation is defined by the relative orientations of dislocation line and Burger's vector. For an edge, they are perpendicular, whereas for a screw, they are parallel. They are neither parallel nor perpendicular for a mixed dislocation. Also, even though a dislocation changes direction and nature within a crystal (e.g. from edge to mixed to screw), the Burger's vector will be the same at all points along its line.

- 1.15** Draw an orthorhombic unit cell and within that cell a $[1\ 2\ 1]$ direction. Take the coordinate system as shown below, with origin point as O .



[6 marks : 2026]

Solution:

The following outlines the methodical procedure for sketching the crystallographic direction $[1\ 2\ 1]$ within an orthorhombic unit cell.

1. PROPERTIES OF THE ORTHORHOMBIC UNIT CELL

Before proceeding with the vector placement, the geometric boundaries of the unit cell must be defined. In an orthorhombic crystal system, the lattice parameters are governed by the following constraints:

- The edge lengths are strictly unequal, such that $a \neq b \neq c$.
- The interaxial angles form mutually orthogonal planes, satisfying the condition $\alpha = \beta = \gamma = 90^\circ$.

2. ANALYZING AND REDUCING THE DIRECTION

To graphically represent the vector such that it is entirely bounded by a single standard unit cell, the given Miller indices must be normalized. The given indices are $u = 1$, $v = 2$, and $w = -1$. Normalization is achieved by dividing each index by the largest absolute magnitude present in the set, which is 2.

$$\begin{aligned}x &= \frac{1}{2}, \\y &= 1, \\z &= -\frac{1}{2}\end{aligned}$$

3. ORIGIN SHIFT

Because the projected z-coordinate yields a negative value ($-1/2$), initiating the vector at the standard lattice origin $O(0, 0, 0)$ would force the vector to terminate outside the established unit cell boundaries. Consequently, the origin must be shifted by one positive unit along the z-axis. The translated origin, denoted as O' , is therefore situated at the coordinates $(0, 0, 1)$.

4. DETERMINING VECTOR COORDINATES

The spatial coordinates for the vector's tail and head, relative to the primary coordinate axes, are calculated by incorporating the normalized projections from the shifted origin's perspective.

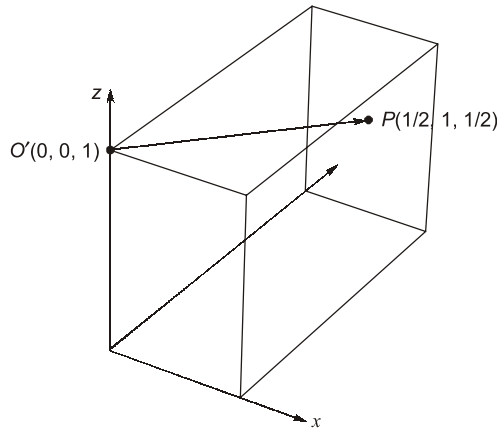
- Tail of Vector (Start): Locates exactly at the shifted origin, $O' (0, 0, 1)$.
- Head of Vector (End): Determined by adding the reduced projections to the starting coordinates:

$$x = 0 + \frac{1}{2} = \frac{1}{2}$$

$$y = 0 + 1 = 1$$

$$z = 1 - \frac{1}{2} = \frac{1}{2}$$

This places the final end point, P , at $(1/2, 1, 1/2)$.



1.16 For a Ceramic compound, mention the two characteristics of the component ions that determine the crystal structure. Further, demonstrate that the minimum cation-to-anion radius ratio for a coordination number of 8 is 0.732. Assume a cubic unit cell. Show the various steps along with the schematic diagram.

[10 marks : 2026]

Solution:

Characteristics Determining Crystal Structure:

The crystal structure of a ceramic compound (typically composed of metallic cations and non-metallic anions) is primarily determined by two characteristics of its component ions:

1. **Magnitude of the Electrical Charge:** The crystal must remain electrically neutral. The ratio of cations to anions must exactly balance the total positive charge with the total negative charge, dictating the chemical formula and overall stoichiometry (e.g., CaF_2 requires two fluoride anions for every calcium cation).
2. **Relative Sizes of the Ions (Cation-to-Anion Radius Ratio):** Metallic elements lose electrons to become cations, making them generally smaller than the non-metallic anions. The structure aims to maximize the number of oppositely charged nearest neighbors (the Coordination Number). This geometrical packing is dictated by the ratio of the cation radius (r_c) to the anion radius (r_a).

Minimum Radius Ratio for a Coordination Number of 8:

For a coordination number (CN) of 8, the cation sits in a cubic interstitial site, surrounded by 8 anions located at the corners of a simple cube. We aim to find the minimum r_c/r_a ratio required for this stable configuration.

Assumptions for the "minimum" radius ratio:

- The anions at the corners just touch each other along the edge of the unit cell cube.
- The cation in the center simultaneously touches all 8 corner anions along the body diagonals of the cube.

Mathematical Derivation:

1. Let r_c be the radius of the cation and r_a be the radius of the anion.
2. Let a be the edge length of the cubic unit cell.
3. Because the anions touch along the edge of the cube, the edge length is:

$$a = 2r_a \quad \dots(i)$$

4. The cation sits at the very center of the body diagonal. The total length of the body diagonal spans the radius of one corner anion, the full diameter of the cation, and the radius of the opposite corner anion:

$$\text{Body Diagonal} = r_a + 2r_c + r_a = 2(r_a + r_c) \quad \dots(\text{ii})$$

5. Geometrically, the length of a body diagonal in a cube with edge length a is calculated using the Pythagorean theorem in 3D:

$$\text{Body Diagonal} = \sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = a\sqrt{3} \quad \dots(\text{iii})$$

6. Equating the two expressions for the body diagonal:

$$2(r_a + r_c) = a\sqrt{3} \quad \dots(\text{iv})$$

7. Substitute $a = 2r_a$ into the equation

$$2(r_a + r_c) = (2r_a)\sqrt{3} \quad \dots(\text{v})$$

8. Divide both sides by $2r_a$ to isolate the ratio:

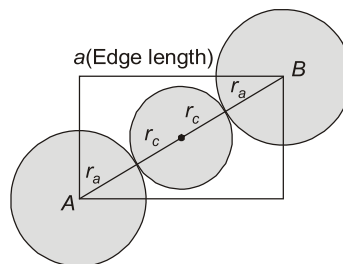
$$\frac{r_a + r_c}{r_a} = \sqrt{3}$$

$$1 + \frac{r_c}{r_a} = \sqrt{3}$$

$$\frac{r_c}{r_a} = \sqrt{3} - 1 \approx 1.732 - 1$$

9. Final result for the minimum radius ratio:

$$\frac{r_c}{r_a} = 0.732 \quad \dots(\text{vi})$$



$$\text{Length of body diagonal } AB = a\sqrt{3}$$

Schematic of the cross-section along the diagonal plane of a cubic unit cell. It shows the central cation touching the corner anions along the body diagonal.

2. Dielectric Properties of Materials

- 2.1** Calculate the polarization induced in Al_2O_3 with dielectric constant $\epsilon_r = 8$, when it is placed between two plates of a parallel plate capacitor separated by 1 mm. Potential difference between the two plates is 10 V.

[12 marks : 2002]

Solution:

$$d = 1 \text{ mm} = 10^{-3} \text{ m}, V = 10 \text{ V}, \epsilon_r = 8$$

$$\text{Electric field} = \frac{\text{potential difference}}{\text{distance}} = V/d$$

$$= \frac{10}{1 \text{ mm}} = 10 \times 10^3 \text{ V/m}$$

