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Topicwise
Conventional Practice Book

Electrical Engineering
PAPER-I

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ESE 2027 Main Examination

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CONTENTS

1 Electrical Materials

1. Crystallography

Level-1

1.1 **Derive atomic packing factor for face centered cubic structure?** **(10 Marks)**

Solution:

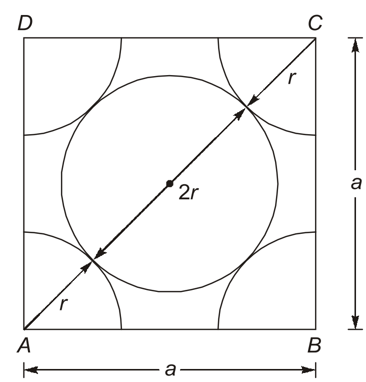
In FCC structure, one atom lies at each corner of the cube. In addition to that one atom lies at the centre of each face.

So effective number of atoms per unit cell will be

$$N = \frac{1}{8}\times 8 + \frac{1}{2}\times 6 = 4$$

$\therefore$ $$N = 4$$

In 2-dimension FCC structure can be drawn as shown in figure.



$\therefore$ atomic radius of FCC is r.

$$AC^2 = AB^2 + BC^2$$

$\therefore$ $$(r + 2r + r)^2 = a^2 + a^2$$

$\therefore$ $$16r^2 = 2a^2$$

$\therefore$ $$r = \frac{\sqrt{2}a}{4}$$

Calculation of APF for FCC :

$$\text{APF} = \frac{\text{Sum of atomic volume in a unit cell}}{\text{Volume of a unit cell}}$$

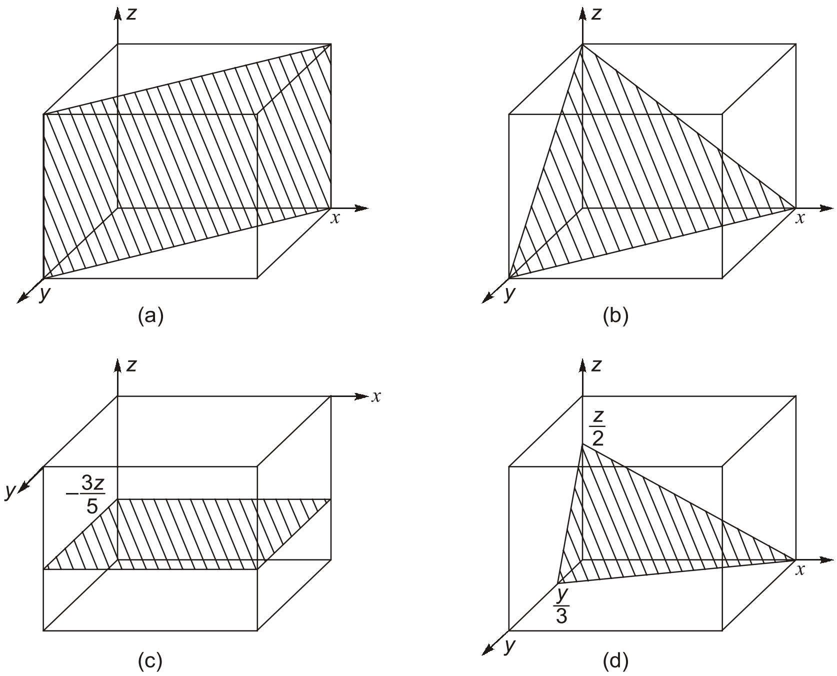
$$= \frac{N\times\frac{4}{3}\pi r^3}{a^3}$$

$$= \frac{4\times\frac{4}{3}\pi\left(\frac{\sqrt{2}a}{4}\right)^3}{a^3}$$

APF = 0.74

$\therefore$ Atomic Packing Factor = 0.74 or 74%.

1.2 Find Miller indices of the following crystal structures? (10 Marks)



Solution:

(a)

Intercepts :	x	y	z
Expressed in terms of unit cell dimension :	1	1	∞
Reciprocal :	1	1	0
Reduce reciprocal to smallest integer :	1	1	0

$\therefore$ Miller indices = [1 1 0]

(b)

Intercepts :	x	y	z
Expressed in terms of unit cell dimension :	1	1	1
Reciprocal :	1	1	1
Reduce reciprocal to smallest integer :	1	1	1

$\therefore$ Miller indices = [1 1 1]

(c)

Intercepts :	∞	∞	$\frac{-3}{5}z$
Expressed in terms of unit cell dimension :	∞	∞	$\frac{-3}{5}$
Reciprocal :	0	0	$\frac{-5}{3}$
Reduce reciprocal to smallest integer :	0	0	–5

$\therefore$ Miller indices = $[0\ \ 0\ \ \bar{5}]$

(d)

Intercepts :	x	$\frac{y}{3}$	$\frac{z}{2}$

Expressed in terms of unit cell dimension :	1	$\frac{1}{3}$	$\frac{1}{2}$
Reciprocal :	1	3	2
Reduce reciprocal to smallest integer :	1	3	2

$\therefore$ Miller indices = [1 3 2]

1.3 (i) Derive relation for atomic radius of unit cell for BCC crystal system and FCC crystal system.

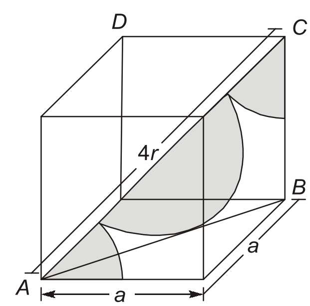
(ii) Enumerate different type of physical properties which get affected by structural imperfection in a crystal. Explain briefly about different types of point defects and line defects.

(12 Marks)

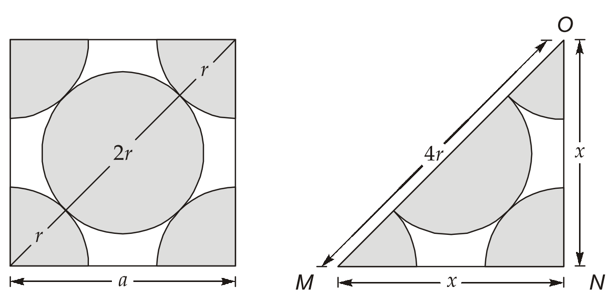
Solution:

(i) In body centered cubic structure, the atoms touch each other along the diagonal of the cube as shown in figure. Therefore, the length of the diagonal of cube is $4r$.

$$AC = \sqrt{AB^2 + BC^2}$$
$$AB = \sqrt{x^2 + x^2} = x\sqrt{2}$$
$$BC = x \qquad (\text{Let } a = x)$$
$$AC = 4r = \sqrt{(x\sqrt{2})^2 + x^2} = x\sqrt{3}$$
$$r = \frac{x\sqrt{3}}{4}$$
$$x = \frac{4r}{\sqrt{3}}$$



In face centered cubic structure, the atoms touch each other along the diagonal of any face of the cube as shown in figure. The length of the diagonal of the face = $4r$ as shown in figure below,



(a) (b)

$$MO^2 = MN^2 + ON^2$$
$$(4r)^2 = x^2 + x^2 = 2x^2$$

or, $$x = \frac{4r}{\sqrt{2}}$$

or, $$r = \frac{x\sqrt{2}}{4}$$

or, $$r^2 = \frac{2x^2}{16} = \frac{x^2}{8}$$

$$r = \frac{x}{2\sqrt{2}}$$

So, $$x = 2\sqrt{2}r$$

(ii) Imperfections result due to deviation from an orderly periodic array. The study of defect is especially useful when studying the properties of materials which are structure sensitive.

This effects the properties like fracture strength plasticity, electrical resistivity and thermal conduction.

Point Defects:

A point defect is a very localized defect in the regularity of a lattice and usually does not spread over more than one or two lattice spacing. The structural point defect can be classified into three types, i.e., vacancies, interstitial, and impurities.

- Vacancies defect: In vacancies defect, there may be missing atom in a crystal. A vacancy may result because of imperfect packing during original crystallization or may arise from thermal vibrations of atoms at elevated temperature.
- Interstitial defect: In interstitial defect, an atom occupies a definite position in the lattice such that it is normally not occupied in a perfect crystal. The position occupied by a normal atom or a foreign atom lies between the atoms of the ideal crystal.
- Impurity defect: In impurity defect, foreign atoms are introduced into a crystal lattice either as an interstitial atom or as a substitution atom. Therefore defects may be classified as interstitial impurity defect and substitution impurity defect.

Line Defects:

Line defects are basically one-dimensional defects or dislocations. Due to dislocations, lattice distortions are centered around a line. Dislocations are produced due to solidification of the crystalline solids by vacancy condensation or by atomic mismatch in the solid solutions. Dislocations are mainly of two types i.e., the edge dislocation and the screw dislocation.

- Edge dislocation: In edge dislocation an extra half plane of atoms is inserted in a crystal either above or below the slip plane. Edge dislocation may be either positive or negative.
- Screw dislocation: The screw dislocation can be created in a perfect crystal by applying upward or downward shear stresses to the regions of perfect crystal, which have been separated by a cutting plane. Screw dislocation may be either positive or negative.

1.4 When an NaCl crystal is subjected to an electric field of 1000 V/m, the resulting polarization is 4.3×10^{-8} C/m^2. Calculate the relative permittivity of NaCl.

(10 Marks)

Solution:

Polarization,
$$P = \varepsilon_0(\varepsilon_r - 1)E$$

$$\varepsilon_r = 1 + \frac{P}{\varepsilon_0 E} = 1 + \frac{4.3 \times 10^{-8}\,\text{C/m}^2}{(8.854 \times 10^{-12}\,\text{F/m})\,(1000\,\text{V/m})}$$

$$\varepsilon_r \approx 5.86$$

Level-2

1.5 The copper crystal has FCC unit cell configuration. If radius of Cu atom is 0.148 nm and atomic mass of Cu is 63.5 gm mol^{-1} then calculate atomic packing fraction (APF), the atomic concentration in a unit cell and density of Cu atom in gcm^{-3}.
(Take Avogadro number : 6.023×10^{23} mol^{-1})

(15 Marks)

Solution:

For the given material (Cu),

$$\text{Atomic packing fraction, (APF)} = \frac{\text{(No. of atom in unit cell)} \times \text{Volume of atom}}{\text{Volume of unit cell}}$$

$$\text{For face centred cubic unit cell} = 4$$

$$\text{Volume of an atom} = \frac{4}{3}\pi R^3$$

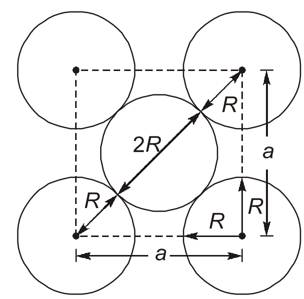
$$\text{Volume of unit cell} = a^3$$

Where,

R is radius of copper atom

a is the cube side length

$$\text{APF} = \frac{4 \times \frac{4}{3}\pi R^3}{a^3} = \frac{\frac{4^2}{3}\pi R^3}{\left(2\sqrt{2}R\right)^3}$$



Face section of unit cell,

As, $$a = 2\sqrt{2}R$$

$$\text{APF} = \frac{4^2\pi}{3\left(2\sqrt{2}\right)^3} = 0.7404$$

Assuming there are x atoms in unit cell,

$$\text{Atomic concentration, } n_{at} = \frac{\text{No. of atoms in unit cell}}{\text{Volume of unit cell}} = \frac{x}{a^3}$$

$$n_{at} = \frac{4}{\left(2\sqrt{2}R\right)^3} = \frac{4}{\left(2\sqrt{2} \times 0.148 \times 10^{-9}\right)^3}$$

$$= \frac{4}{(4.186 \times 10^{-10})^3} = 5.453 \times 10^{28}\ \text{m}^{-3} = 5.453 \times 10^{22}\ \text{cm}^{-3}$$

$$\text{The density of copper, } \rho = \frac{\text{Mass of all atoms in unit cell}}{\text{Volume of unit cell}}$$

$$= \frac{x\left(\frac{M_{at}}{N_A}\right)}{a^3} \qquad \text{(Where } M_{at} \text{ is atomic mass of Cu)}$$

$$\therefore \qquad \rho = \frac{n_{at}M_{at}}{N_A} = \frac{5.453 \times 10^{22} \times 63.5\,\text{gmmol}^{-1}}{6.023 \times 10^{23}\,\text{mol}^{-1}}$$

$$= 5.749\ \text{gm cm}^{-3}$$

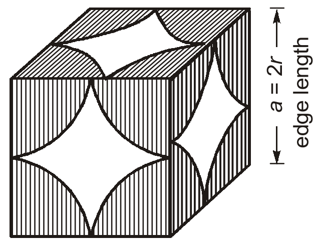
1.6 What are the types of cubic crystal structure? Derive the atomic packing factor of all the cubic crystal structures.

(20 Marks)

Solution:

There are three types of cubic crystal structures:

- Simple Cubic Crystal structure (SCC):
  In this structure, there is one Lattice point at each of the eight corners of the unit cell. It has a coordination number of six.

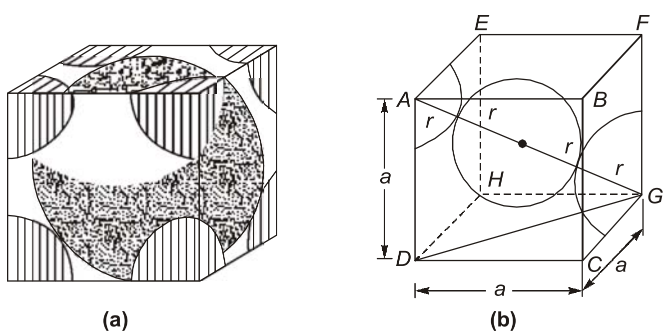


Atomic Packing Factor (APF) of the crystal structure is defined as the ratio of total volume of the atoms per unit cell to the volume of the unit cell. It is also known as packing efficiency (η)

Here, $$\text{APF} = \frac{1\times(\pi/6)a^3}{a^3} = \frac{\pi}{6} = 0.524$$

$\therefore$ $$\%\ \text{APF} = 52.4\%\ \text{filled}$$

- Body Centered Cubic structure (BCC):
  In this structure, in an unit cell there are eight atoms at corners and another atom is at the body center. It has a coordination number of eight.



Body centered cubic structure

From the figure, $$r = \frac{a\sqrt{3}}{4}$$

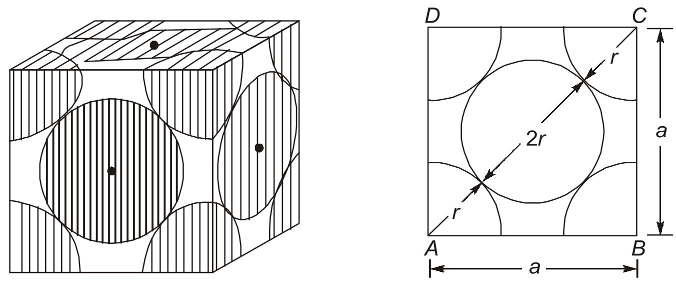
and for BCC, $$N = 2$$

Now, $$(\text{APF})_{\text{BCC}} = \frac{N \times \text{Volume of each sphere}}{\text{Total volume of each cell}}$$

$$= \frac{2\times\frac{4}{3}\pi\left(\frac{a\sqrt{3}}{4}\right)^3}{a^3} = \frac{\frac{2\times\pi\sqrt{3}}{16}a^3}{a^3} = \frac{\pi\sqrt{3}}{8} = 0.68$$

$\therefore$ $$\%\ \text{APF} = 68\%\ \text{filled}$$

- Face Centered Cubic structure (FCC):
  In this structure, one atom lies at each corner of the cube in addition to one atom at the center of each face. The coordination number of FCC structure is (4 + 4 + 4 = 12)



Face centered cubic structure

From the figure,

$$\text{Radius of FCC} = r = \frac{a\sqrt{2}}{4}$$

and for FCC, $N = 4$

Now,

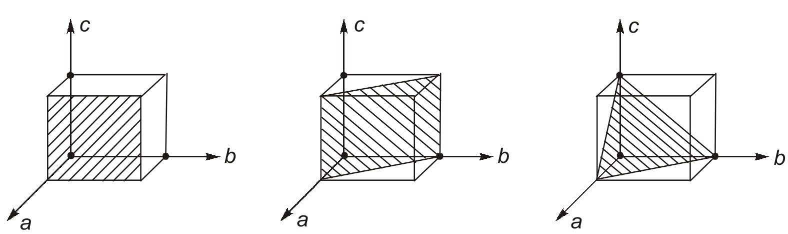
$$(\text{APF})_{\text{FCC}} = \frac{N \times \text{Volume of each sphere}}{\text{Total volume of each cell}}$$

$$= \frac{4 \times \frac{4}{3}\pi \times \left(\frac{a\sqrt{2}}{4}\right)^3}{a^3} = \frac{4 \times \frac{\sqrt{2}\pi a^3}{24}}{a^3} = \frac{\sqrt{2}\pi}{6} = \frac{\pi}{3\sqrt{2}} = 0.74$$

$\therefore$ % APF = 74 % filled

1.7 Draw sketches illustrating a (100) plane, a (110) plane, and a (111) plane in a cubic unit cell. How many equivalent (100) planes are there in a cubic crystal? A material has a face-centered cubic structure with an ionic radius of 1.06 Å. Calculate the inter planar separation for (111) planes. (15 Marks)

Solution:



(100) Plane (110) Plane (111) Plane

There are six equivalent (100) planes in a cubic crystal.

$\Rightarrow$ In a face – centerd cubic structure.

$$a\sqrt{2} = 4r$$

where a = side of cube

r = ionic radius

Given : r = 1.06 Å

$$a = \frac{4r}{\sqrt{2}} = \frac{4 \times 1.06}{\sqrt{2}} = 2.998 \text{ Å}$$

Inter planar separation for (111) planes is

$$d = \frac{a}{\sqrt{3}} = \frac{2.998}{\sqrt{3}} = 1.731 \text{ Å}$$

2. Dielectric Properties of Materials

Level-1

2.1 A long narrow rod has an atomic density 4 × 10^{28} atoms/m^3. Each atom has a polarizability of 2 × 10^{-40} Farad m^2. Find the internal electric field when axial field of 2 V/m is applied?

(10 Marks)

Solution:

$$N = 4 \times 10^{28} \text{ atoms/m}^3$$

$$\alpha = 2 \times 10^{-40} \text{ farad-m}^2$$

$$E = 2 \text{ V/m}$$

Now,
$$E_i = E + \frac{P}{3\varepsilon_0} = E + \frac{N\alpha E_i}{3\varepsilon_0}$$

$\therefore$
$$E_i = \frac{E}{1 - \dfrac{N\alpha}{3\varepsilon_0}} = \frac{2}{1 - \dfrac{4 \times 10^{28} \times 2 \times 10^{-40}}{3 \times 8.85 \times 10^{-12}}}$$

$\therefore$
$$E_i = 2.86 \text{ V/m}$$

$\therefore$ Internal electric field = 2.86 V/m.

2.2 (a) The following data refers to a dielectric material ε_r = 3.95, n^2 = 2.5, where n is the index of refraction. Calculate ratio of electronic and ionic polarizability for this material? (10 Marks)

(b) A solid specimen of dielectric has a dielectric constant 4 and tan δ = 0.002 at a frequency of 75 Hz. If it is subjected to an alternating field of 60 kV/cm. Calculate the heat generated in the specimen due to dielectric loss?

(5 Marks)

Solution:

(a)
$$\varepsilon_r = 3.95 \text{ , } n^2 = 2.5$$

CM equation is

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N(\alpha_e + \alpha_i)}{3\varepsilon_0}$$

$\therefore$
$$\frac{3.95 - 1}{3.95 + 2} = \frac{N(\alpha_e + \alpha_i)}{3\varepsilon_0}$$

$\therefore$
$$\frac{N(\alpha_e + \alpha_i)}{3\varepsilon_0} = \frac{2.95}{5.95} \quad \text{...(1)}$$

and at optical frequencies, $\alpha_i = 0$

$\therefore$
$$\alpha = \alpha_e \text{ and } \varepsilon_r = n^2$$

$\therefore$
$$\frac{n^2 - 1}{n^2 + 2} = \frac{N(\alpha_e)}{3\varepsilon_0}$$

$\therefore$
$$\frac{2.5 - 1}{2.5 + 2} = \frac{N(\alpha_e)}{3\varepsilon_0}$$

$$\therefore \quad \frac{N(\alpha_e)}{3\varepsilon_0} = \frac{1.5}{4.5} \quad \text{...(2)}$$

Dividing (1) by (2)

$$\therefore \quad \frac{\alpha_e + \alpha_i}{\alpha_e} = \frac{2.95}{5.95} \times \frac{4.5}{1.5}$$

$$\therefore \quad 1 + \frac{\alpha_i}{\alpha_e} = 1.4873$$

$$\therefore \quad \frac{\alpha_i}{\alpha_e} = 0.4873$$

$$\therefore \quad \boxed{\frac{\alpha_e}{\alpha_i} = 2.051}$$

(b) $\varepsilon_r = 4$, $\tan\delta = 0.002$, $f = 75$ Hz, $E = 60 \times 10^3$ V/cm

$$\therefore \quad w = \frac{\varepsilon_r \cdot E^2 \cdot f \cdot \tan\delta}{1.8 \times 10^{12}} \text{ watt/cm}^3$$

$$= \frac{4 \times (60 \times 10^3)^2 \times 75 \times 0.002}{1.8 \times 10^{12}}$$

$$\therefore \quad w = 1.2 \text{ mW/cm}^3$$

$$\therefore \quad \boxed{\text{Heat generated} = 1.2 \text{ mW/cm}^3}$$

2.3 (a) The dielectric constant of Lithium, measured at 0°C and 1 atmosphere is $\varepsilon_r = 1.0000784$. Under these conditions the gas contains 3×10^{25} atoms/m^3 then calculate radius of electron cloud? (5 Marks)

(b) Also calculate the displacement x when a lithium atom is subjected to a field of 10^6 V/m? (5 Marks)

Solution:

(a) Given :

$$\varepsilon_r = 1.0000784, \; N = 3 \times 10^{25} \text{ atoms/m}^3$$

We know that, $\varepsilon_r = 1 + 4\pi NR^3$

$$\therefore \quad R = \sqrt[3]{\frac{\varepsilon_r - 1}{4\pi N}}$$

$$\therefore \quad R = 5.92 \times 10^{-11} \text{ m}$$

$\therefore$ Radius of electron cloud $= 5.92 \times 10^{-11}$ m

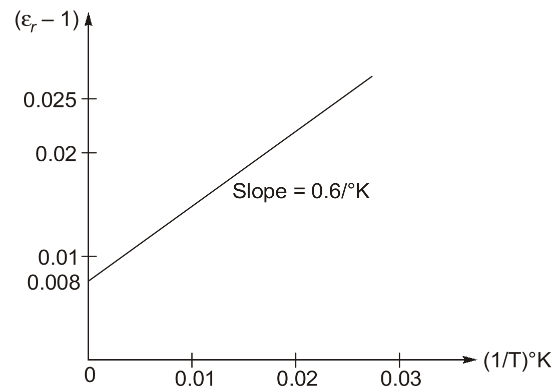
(b) As given atom is lithium atom $Z_e = 3$ and $E = 10^6$ V/m

We know that, $$x = \frac{4\pi\varepsilon_0 R^3}{Z_e} E = \frac{4\pi\varepsilon_0 \times (5.92 \times 10^{-11})^3}{3} \times 10^6$$

$$x = 7.713 \times 10^{-36} \text{ m}$$

$\therefore$ Electron cloud will shift by 7.713×10^{-36} m w.r.t. nucleous.

2.4 The variation of dielectric constant of a material of a function of temperature is plotted in below figure. Calculate the permanent dipole moment and the polarizability of molecules. Number of molecules per m^3 are 1.5×10^{25}? (10 Marks)



Solution:

Given : $N = 1.5 \times 10^{25}$ molecules/m^3

Total polarization in dielectric is

$$P_T = P_e + P_i + P_o$$

$$\varepsilon_0(\varepsilon_r - 1)E = N\alpha_e E + N\alpha_i E + \frac{NP_P^2}{3KT}E$$

$$\therefore \quad \underbrace{\varepsilon_0(\varepsilon_r - 1)}_{y} = \underbrace{N(\alpha_e + \alpha_i)}_{c} + \underbrace{\frac{NP_P^2}{3K}}_{m} \cdot \underbrace{\frac{1}{T}}_{x}$$

Comparing this with the graph given in exam

$$\therefore \quad N(\alpha_e + \alpha_i) = 0.008\varepsilon_0 \text{ and } \frac{NP_P^2}{3K} = 0.6\varepsilon_0$$

$$\therefore \quad \alpha_e + \alpha_i = \frac{0.008\varepsilon_0}{N}$$

$$\therefore \quad \alpha_e + \alpha_i = 4.72 \times 10^{-39} \text{ F-m}^2$$

$\therefore$ Polarizability is 4.72×10^{-39} F-m^2

and $$\frac{NP_P^2}{3K} = 0.6\varepsilon_0$$

$$\therefore \quad P_P = \sqrt{\frac{0.6\varepsilon_0 \times 3K}{N}} = 3.83 \times 10^{-30} \text{ c-m}$$

$\therefore$ Permanent dipole Moment $= 3.83 \times 10^{-30}$ c-m

Level-2

2.5 (a) What is Polarization? (10 Marks)

(b) Derive relationship between $\bar{P}$ and $\bar{E}$ in terms of dielectric susceptibility? (5 Marks)

Solution:

(a) In dielectric materials the bounded electrons are predominant.

Under the application of external field the bound electrons of an atom are displaced such that centroid of the electron cloud is separated from the centroid of the nucleous. The atom is then said to be polarized thereby creating an electric dipole. This phenomenon is called Polarization.