

ESE 2027

Main Examination

UPSC ENGINEERING SERVICES EXAMINATION

Topicwise
**Conventional
Practice Book**

Mechanical Engineering

PAPER-II





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**Main Examination • Conventional Practice Book :
Mechanical Engineering PAPER-II**

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1

Mechanisms & Machines

1. Simple Mechanisms

Level-1

1.1 What do you understand by mechanisms and machines? Explain Geneva drive and Toggle mechanism with their application.

(4+4 = 8 Marks)

Solution:

A mechanism is a combination of rigid bodies (resistant or restrained body) so shaped and connected that they move upon each other with definite relative motion.

Example : Single slider crank mechanism.

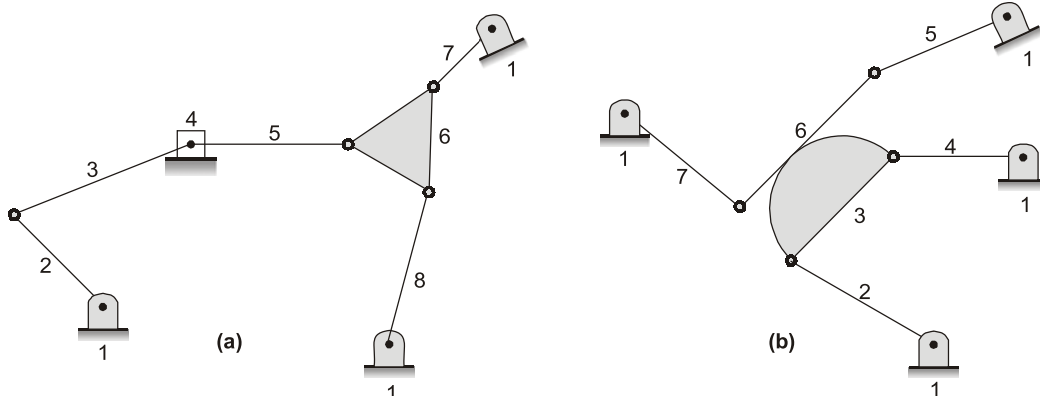
A machine is a mechanism or a collection of mechanisms which transmit force from the source of power to the resistance (load) to be overcome and thus perform useful mechanical work.

Example : Internal combustion engine.

Geneva Drive : It is a gear mechanism that translates a continuous rotation into an intermittent rotary motion. Its application is in movie projectors : the film does not run continuously through the projector. Instead, the film is advanced frame by frame, each frame standing still in front of the lens for 1/24 of a second. (and being exposed twice in that time, resulting in a frequency of 48 Hz.)

Toggle Mechanism : The mechanism used to overcome a large resistance of a member with a small driving force is known as snap action or toggle mechanism. Its application is in stone crushers, embossing presses, switches, etc.

1.2 Determine the degree of freedom of the following mechanisms.



(4+4 = 8 Marks)

Solution:

(a) The DOF of the mechanism is found by Gruebler's criterion,

$$\text{Total number of links} = 8$$

$$\text{Number of pairs with 1 DOF} = 10$$

(At the slider - one sliding pair and two turning pairs)

$$\text{DOF} = 3(N - 1) - 2J - H = 3(8 - 1) - 2(10) - 0 = 1$$

(b) Total number of links = 7

$$\text{Number of lower pair} = 8$$

$$\text{Number of higher pair} = 1$$

$$\text{DOF} = 3(7 - 1) - 2(8) - 1 = 3(6) - 16 - 1 = 1$$

1.3 Write down the three positions of correct gearing for Ackermann steering gear. Also, derive the expression :

$$\tan \alpha = \frac{\sin \phi - \sin \theta}{\cos \theta + \cos \phi - 2}$$

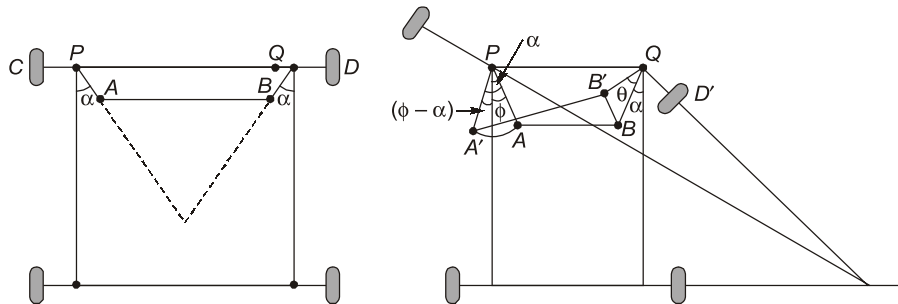
Where, θ and ϕ = angles turned by the stub axle and α = inclination of the track arms to the longitudinal axis of vehicle.

(6 Marks)

Solution:

For the Ackermann gear, three positions of correct gearing are :

- When the vehicle moves straight.
- When the vehicle moves at a correct angle to the right, and
- When the vehicle moves at a correct angle to the left



The angles θ , ϕ and α are taken as per the question.

Now from the figure,

$$\text{Projection of } BB' \text{ on } PQ = \text{Projection of } AA' \text{ on } PQ$$

$$QB[\sin(\alpha + \theta) - \sin \alpha] = PA[\sin \alpha + \sin(\phi - \alpha)]$$

$$\sin(\alpha + \theta) - \sin \alpha = \sin \alpha + (\sin \phi - \alpha)$$

$$[\because PA = QB]$$

$$\Rightarrow (\sin \alpha \cos \theta + \cos \alpha \sin \theta) - \sin \alpha = \sin \alpha + (\sin \phi \cdot \cos \alpha - \cos \phi \cdot \sin \alpha)$$

$$\sin \alpha (\cos \theta + \cos \phi - 2) = \cos \alpha (\sin \phi - \sin \theta)$$

$$\therefore \frac{\sin \alpha}{\cos \alpha} = \frac{\sin \phi - \sin \theta}{\cos \theta + \cos \phi - 2}$$

$$\therefore \tan \alpha = \frac{\sin \phi - \sin \theta}{\cos \theta + \cos \phi - 2}$$

where θ and ϕ are the values of angles for the correct gearing.

Level-2

- 1.4** What is a quick-return mechanism? Give its types and applications. How is the ratio of time of cutting stroke to return stroke calculated for a slotted lever and crank type of quick-return mechanism? Explain with the help of a neat sketch.

(15 Marks)

Solution:

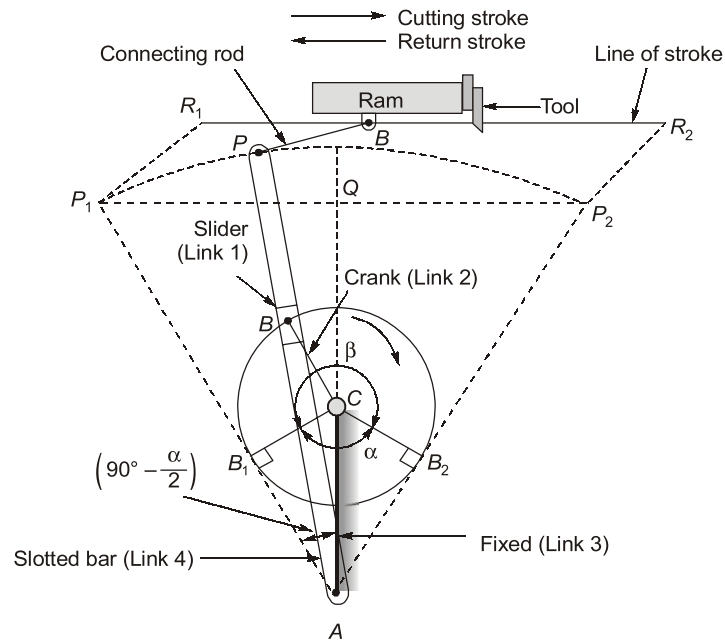
A quick return mechanism is a mechanism which converts circular motion (rotating motion following a circular path) into reciprocating motion (repetitive back-and-forth or to-and-fro linear motion) in presses and shaping machines.

There are three types of quick return mechanism

1. Hydraulic shaper drive
2. Crank and slotted lever mechanism
3. Whitworth mechanism

Following are the applications of quick-return mechanism:

- Shaper
- Power-driven saw
- Revolver mechanisms
- Screw press
- Mechanical actuator



Crank and slotted lever quick return motion mechanism

In the extreme positions, AP_1 and AP_2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position CB_1 to CB_2 (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position CB_2 and CB_1 (or through angle α) in the clockwise direction. Since the crank has uniform angular speed, therefore,

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} \text{ or } \frac{360^\circ - \alpha}{\alpha}$$

1.5 Derive an expression for the ratio of angular velocities of the shafts of a Hooke's joint.

(16 Marks)

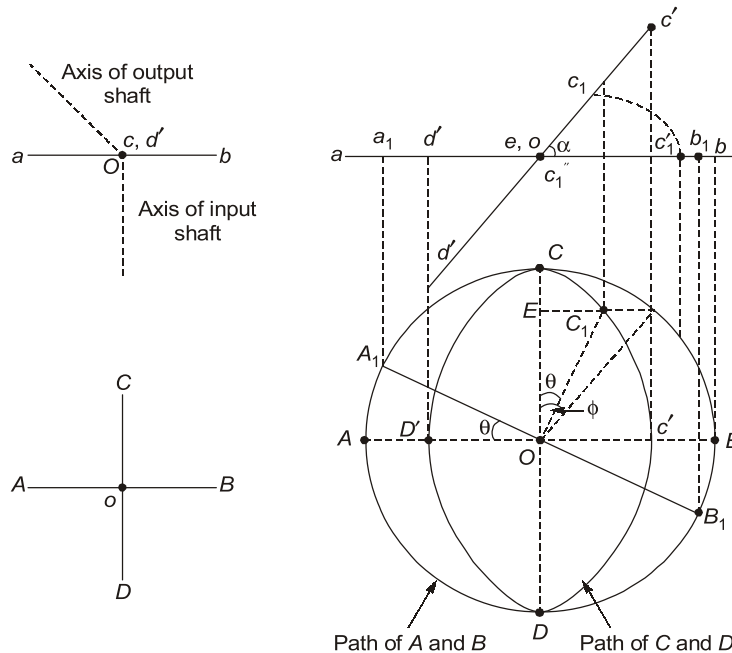
Solution:

Let two horizontal shafts, the axes of which are at an angle α , be connected by Hooke's joint.

If the joint is viewed along the axis of the shaft 1, the fork ends of this shaft will be A and B as shown in figure I. C and D are the positions assumed by the fork ends of the shaft 2. The axis of the shaft 1 is along the perpendicular to the plane of paper at O and that of the shaft 2 along OA .

When viewed from top, c and d , projections of C and D coincide with that of O whereas a and b remain unchanged.

As the shaft 1 is rotated, its fork ends A and B are rotated in a circle (figure II). However, the fork ends C and D of the shaft 2 will move along the path of an ellipse, if viewed along the axis of the shaft 1. In the top view, the motion of the fork ends of the shaft 1 is along the line ab whereas that of the shaft 2 is on a line $c'd'$ at an angle of α and ab .

**Figure I****Figure II**

Let the shaft 1 rotate through an angle θ so that fork ends assume the position A_1 and B_1 . Now, the angle moved by the shaft 2 would also be θ when it is viewed along its own axis. Let ϕ be the angle turned by shaft 2.

Now, from figure,

$$\frac{\tan \phi}{\tan \theta} = \frac{EC'_1 / EO}{EC_1 / EO} = \frac{EC'_1}{EC_1} = \frac{ec'_1}{ec''_1} \quad (\text{figure II top view})$$

$$\frac{ec_1}{ec''_1} = \frac{1}{ec'_1 / ec_1} = \frac{1}{\cos \alpha} \quad (\text{from the II figure})$$

$$\tan \theta = \tan \phi \cdot \cos \alpha$$

Let,

$$\omega_1 = \text{angular velocity ratio of driving shaft} = \frac{d\theta}{dt}$$

$$\omega_2 = \text{angular velocity ratio of driven shaft} = \frac{d\phi}{dt}$$

Now, differentiating with respect to time t ,

$$\begin{aligned}
 \sec^2 \theta \cdot \frac{d\theta}{dt} &= \cos \alpha \cdot \sec^2 \phi \cdot \frac{d\phi}{dt} \\
 \therefore \frac{\omega_2}{\omega_1} &= \frac{1}{\cos^2 \theta \cdot \cos \alpha (1 + \tan^2 \phi)} \\
 &= \frac{1}{\cos^2 \theta \cdot \cos \alpha \left(1 + \frac{\tan^2 \theta}{\cos^2 \alpha} \right)} \quad \left(\tan \phi = \frac{\tan \theta}{\cos \alpha} \right) \\
 &= \frac{1}{\cos^2 \theta \cdot \cos \alpha \left(1 + \frac{\sin^2 \theta}{\cos^2 \theta \cdot \cos^2 \alpha} \right)} \\
 &= \frac{\cos^2 \theta \cdot \cos^2 \alpha}{\cos^2 \theta \cdot \cos \alpha (\cos^2 \theta \cos^2 \alpha + \sin^2 \theta)} = \frac{\cos \alpha}{\cos^2 \theta - \cos^2 \theta \sin^2 \alpha + \sin^2 \theta} \\
 \frac{\omega_2}{\omega_1} &= \frac{\cos \alpha}{1 - \sin^2 \alpha \cdot \cos^2 \theta}
 \end{aligned}$$

2. Velocity and Acceleration

Level-1

2.1 What is the corioli's acceleration component? Derive the expression for it.

(10 Marks)

Solution:

Consider the case of slotted lever where slider is performing reciprocating motion inside the lever.

The slider motion from B to F occurs in 3 stages :

- (i) B to D due to rotation of OA
- (ii) D to E outwards velocity of slider.
- (iii) E to F due to acceleration perpendicular to link OA , which is the corioli's acceleration component.

Now, from the figure.

$$\begin{aligned}
 \text{Arc } EF &= \text{Arc } CF - \text{Arc } CE \\
 &= OC \cdot \delta\theta - \text{Arc } BD \quad (\because \text{Arc } CE = \text{Arc } BD) \\
 &= OC \cdot \delta\theta - OB \cdot \delta\theta = (OC - OB) \cdot \delta\theta
 \end{aligned}$$

$$BC \cdot \delta\theta = DE \cdot \delta\theta$$

Now

$$DE = v \cdot dt$$

$\therefore$

$$\omega = \frac{\delta\theta}{\delta t} = \delta\theta = \omega \cdot \delta t$$

Thus,

$$\text{arc } EF = v \cdot dt \cdot \omega \cdot dt = \omega \cdot v (dt)^2 \quad \dots(i)$$

Now, displacement,

$$EF = \frac{1}{2} f_{cc} (dt)^2 \quad \dots(ii)$$

Where f_{cc} is corioli's acceleration for angular displacement EF .

Thus, from equation (i) and (ii)

$$\frac{1}{2} f_{cc} (dt)^2 = \omega v \cdot (dt)^2$$

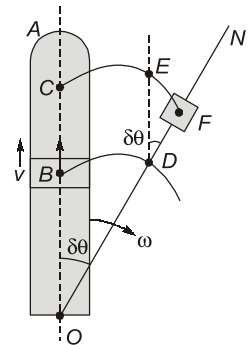
$\therefore$

$$f_{cc} = 2v\omega$$

Where,

ω = angular velocity of slotted lever, and

v = velocity of slider



2.2 What do you mean by instantaneous centre of rotation? Explain the types of instantaneous centres with an example?

(5 Marks)

Solution:

A link or a rigid body as a whole may be considered to be rotating about an imaginary centre or a given centre at a given instant which has zero velocity, then the link is at rest at this point which is known as instantaneous centre of rotation.

Types of instantaneous centres :

- (i) Primary instantaneous centres
- (ii) Secondary instantaneous Centres

Primary instantaneous centres are further divided into fixed and permanent instantaneous centers.

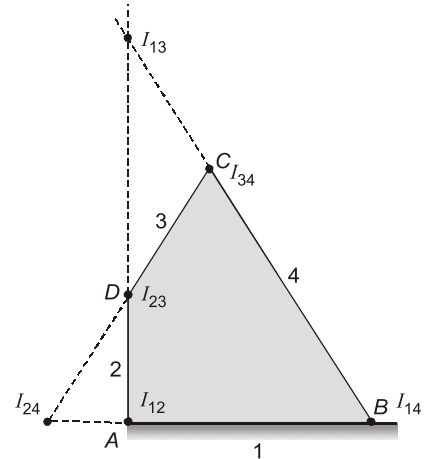
Example :

Let us take a 4 bar mechanism with links AB , BC , CD and DA .

Here the number of instantaneous centres are

$$N = \frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6$$

In the figure, I_{12} and I_{14} are fixed instantaneous centres of rotation, I_{23} and I_{34} are permanent instantaneous centres. Thus, I_{12} , I_{14} , I_{23} and I_{34} are primary instantaneous centres of rotation. Also, I_{13} and I_{24} are secondary instantaneous centres.



Level-2

2.3 A single cylinder horizontal reciprocating engine mechanism has a crank of 8 cm length and connecting rod 36 cm length. The engine speed is 2000 rpm clockwise. Determine the velocity and acceleration of piston when the crank is 315° from inner dead centre. Also determine the angular acceleration of connecting rod and total acceleration of its mid-point. Use relative velocity and acceleration method only.

(15 Marks)

Solution :

As per given data:

Horizontal reciprocating engine mechanism.

Crank, $r = 8$ cm

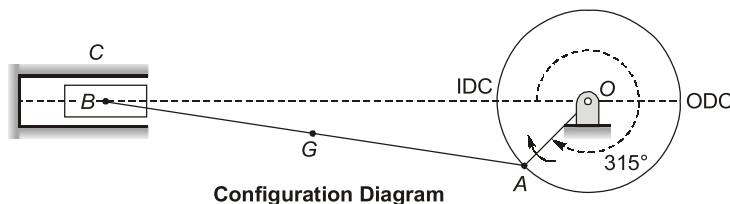
Connecting rod, $l = 36$ cm

Engine speed, $N = 2000$ rpm (CW)

Velocity and acceleration of piston when the crank is 315° from inner dead centre.

Configuration diagram by assuming scale {1 cm = 4 cm}

$OA = 8$ cm ; $AB = 36$ cm



Velocity diagram:

Assuming scale [1 cm = 4 m/s]

Velocity of crank, $OA = r \times \omega = \frac{r \times 2\pi \times N}{60} = \frac{0.08 \times 2\pi \times 2000}{60} = 16.75 \text{ m/s}$

In diagram, $oa = \frac{16.75}{4} = 4.1875 \text{ cm}$

From diagram,

Velocity of piston, $ob = 3.4 \text{ cm} = 3.4 \times 4 \Rightarrow 13.6 \text{ m/s}$

Velocity of piston w.r.t. crank = $ba = 3 \text{ cm} \Rightarrow 3 \times 4 = 12 \text{ m/s}$

Acceleration diagram:

$$\alpha_{\text{crank}} = 0$$

Assume scale 1 cm = 1000 m/s²

Point	w.r.t.	Procedure
A	O	$a_{AO}^r = \frac{V_{AO}^2}{AO} = 3.507 \times 10^3 \text{ m/s}^2$ along $A \rightarrow O$ $a_{AO}^t = AO \times \alpha_{AO} = 0 \perp^{ar}$ to AO
B	A	$a_{BA}^r = \frac{V_{BA}^2}{AB} = \frac{13.6^2}{0.36} = 0.513 \times 10^3 \text{ m/s}^2$ along $B \rightarrow A$ $a_{BA}^t = BA \times \alpha_{BA} = \text{unknown} \perp^{ar}$ to BA
B	C	$a_{BC}^r = \frac{V_{BC}^2}{BC} = 0$ along $B \rightarrow C$ $a_{BC}^t = BC \times \alpha_{BC} = \text{unknown} \perp^{ar}$ to BC

From acceleration diagram

$$o'a' = 3.507 \text{ cm}$$

$$a'x = 0.513 \text{ cm}$$

$$xb' = 2.6 \text{ cm}$$

$$o'b' = 1.55 \text{ cm}$$

$$a'b' = 2.65 \text{ cm}$$

$$o'g' = 2.35 \text{ cm}$$

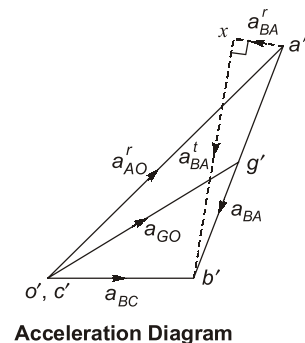
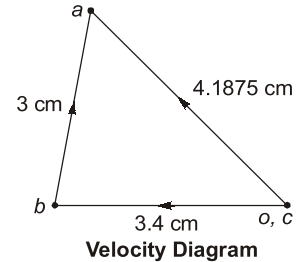
The tangential component of acceleration of connecting rod, $xb' = 2.6 \times 10^3 \text{ m/s}^2 = \alpha_{AB} \times AB$

$$\alpha_{AB} = \frac{2.6 \times 10^3}{0.36} = 7.222 \times 10^3 \text{ rad/s}^2$$

The acceleration of piston, $o'b' = 1.55 \times 10^3 \text{ m/s}^2$.

Total acceleration of connecting rod at mid-point,

$$o'g' = 2.35 \times 10^3 \text{ m/s}^2$$



3. Kinematic and Dynamic Analysis

Level-1

3.1 The crank and the connecting rod of a vertical single cylinder gas engine running at 1800 rpm are 60 mm and 240 mm, respectively. The diameter of the piston is 80 mm and the mass of the reciprocating parts is 1.2 kg. At a point during the power stroke when the piston has moved 20 mm from the top dead centre position, the pressure on the piston is 800 kN/m². Find :

- (i) net force on the piston. (ii) net load on the connecting rod
 (iii) thrust on the sides of cylinder walls (iv) engine speed at which the above values are zero.

(16 Marks)

Solution:

Given :

$$r = 0.06 \text{ m}, \quad l = 0.24 \text{ m}, \quad N = 1800 \text{ rpm}, \quad m = 1.2 \text{ kg}$$

$$n = \frac{0.24}{0.06} = 4, \quad d = 0.08, \quad \omega = \frac{2\pi \times 1800}{60} = 188.5 \text{ rad/s}$$

By drawing the configuration for the given position to some scale, the angle θ is found to be 43.5°

$$\sin \beta = \frac{\sin \theta}{n} = \frac{\sin 43.5}{4} = 0.1721$$

 $\Rightarrow$

$$\beta = \sin^{-1}(0.1721) = 9.91$$

Force due to gas pressure,

$$F_p = \text{Area} \times \text{pressure}$$

$$= \frac{\pi}{4} d^2 \times p = \frac{\pi}{4} (0.08)^2 \times 800 \times 10^3 = 4021 \text{ N}$$

$$\text{Accelerating Force, } F_b = m r \omega^2 \left(\cos \theta + \frac{\cos 2\theta}{n} \right)$$

$$= 1.2 \times 0.06 \times (188.5)^2 \times \left(\cos 43.5 + \frac{\cos 87}{4} \right)$$

$$= 1889 \text{ N}$$

(i) Force on the piston,

$$F = F_p + mg - F_b$$

$$F = 4021 + 1.2 \times 9.81 - 1889$$

$$= 2144 \text{ N}$$

(ii) Net load on the connecting rod,

$$F_c = \frac{F}{\cos \beta} = \frac{2144}{\cos 9.91^\circ} = 2176 \text{ N}$$

(iii) Thrust on the sides of cylinder walls,

$$F_n = F \tan \beta = 2144 \tan 9.91^\circ = 374.57 \text{ N}$$

(iv) The above values are zero at the speed when the force on the piston F is zero

$$F = F_p - m r \omega^2 \left(\cos \theta + \frac{\cos 2\theta}{n} \right) + mg$$

$$0 = 4021 - 1.2 \times 0.06 \omega^2 \left(\cos 43.5 + \frac{\cos 87}{4} \right) + 1.2 \times 9.81$$

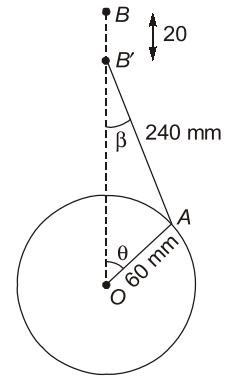
$$0.05317 \omega^2 = 4032.8$$

$$\omega = 75849$$

$$\frac{2\pi N}{60} = 275.4$$

 $\Rightarrow$

$$N = 2630 \text{ rpm}$$

**Level-2**

3.2 A horizontal gas engine running at 240 rpm has a bore of 220 mm and a stroke of 440 mm. The connecting rod is 924 mm long and the reciprocating parts weigh 20 kg. When the crank has turned through an angle of 30° from the inner dead centre, the gas pressures on the cover and the crank sides are 500 kN/m^2 and 60 kN/m^2 respectively. Diameter of the piston rod is 40 mm. Determine :

(i) Turning moment on the crank shaft

(ii) Thrust on the bearings

(iii) Acceleration of the flywheel which has a mass of 8 kg and radius of gyration of 600 mm while the power of the engine is 22 kW. (15 Marks)

Solution:

Given : $r = \frac{0.44}{2} = 0.22 \text{ m}, l = 0.924 \text{ m}, N = 240 \text{ rpm}$

$$m = 20 \text{ kg}, \theta = 30^\circ, n = \frac{l}{r} = \frac{0.924}{0.22} = 4.2$$

Flywheel mass = 8 kg

$$\omega = \frac{2\pi \times 240}{60} = 25.13 \text{ rad/s}$$

$$\therefore \sin \beta = \frac{\sin \theta}{n} = \frac{\sin 30^\circ}{4.2} = 0.119$$

or $\beta = 6.837$

Thus, pressure force on piston, $F_p = p_1 A_1 - p_2 A_2$

$$\Rightarrow F_p = \left[500 \times 10^3 \times \frac{\pi}{4} \times 0.22^2 - 60 \times 10^3 \times \frac{\pi}{4} \times (0.22^2 - 0.04^2) \right]$$



$$\Rightarrow = 19007 - 2206 = 16801 \text{ N}$$

$$\text{Inertia force, } F_b = mf = mr\omega^2 \left(\cos \theta + \frac{\cos 2\theta}{n} \right)$$

$$= 20 \times 0.22 \times (25.13)^2 \times \left(\cos 30^\circ + \frac{\cos 60^\circ}{4.2} \right)$$

$$= 2737.44 \text{ N}$$

$$\text{Piston effort, } F = F_p - F_b = 16801 - 2737.44 = 14063.56 \text{ N}$$

(i) Turning moment, $T = F \cdot \frac{\sin(\theta + \beta)}{\cos \beta} \cdot r$

$$T = \frac{14063.56}{\cos 6.837} \sin(30^\circ + 6.837) \times 0.22 = 1868.25 \text{ Nm}$$

(ii) Thrust on the bearings, $F_r = \frac{F}{\cos \beta} \cdot \cos(\theta + \beta)$

$$F_r = \frac{14063.56}{\cos 6.837} \cos(30 + 6.837) = 11336.55 \text{ N}$$

(iii) Accelerating torque = Turning moment – Resisting torque
Resisting torque can be found from,

$$P = T\omega$$

or, $22 \times 10^3 = T \times 25.13$

$$T = 875.45 \text{ Nm}$$

$$\therefore \text{Accelerating torque} = 1868.25 - 875.45 = 992.8 \text{ Nm}$$

or, $I\alpha = Mk^2\alpha = 992.8$

$$8 \times 0.6^2 \times \alpha = 992.8$$

$$\therefore \text{Acceleration of flywheel, } \alpha = 344.72 \text{ rad/s}^2$$

3.3 Derive the expression for angular velocity and angular acceleration of connecting rod of a single slider crank mechanism.

(15 Marks)

Solution:

Let ω the angular velocity of crank OA , θ be the angle turned by crank from the inner dead centre

Here, $\cos\beta = \frac{A'B_1}{AB_1}$

Here, $AB_1 = l$
 $OA = r$ and $l = nr$

and thus, $A'B_1 = \sqrt{l^2 - (r \sin\theta)^2}$

$$\cos\beta = \frac{\sqrt{n^2 r^2 - r^2 \sin^2 \theta}}{nr} = \frac{\sqrt{n^2 - \sin^2 \theta}}{n} \quad \dots(i)$$

Now, $\sin\beta = \frac{AA'}{l}$

Thus, $AA' = r \cdot \sin\theta$

$$\sin\beta = \frac{\sin\theta}{n}$$

Differentiating with respect to time,

$$\cos\beta \cdot \frac{d\beta}{dt} = \frac{1}{n} \cdot \cos\theta \cdot \frac{d\theta}{dt}$$

$$\Rightarrow \frac{d\beta}{dt} = \omega \cdot \frac{\cos\theta}{n \cos\beta} \quad \left[\text{as } \frac{d\theta}{dt} = \omega \right]$$

$$\Rightarrow \omega_c = \text{angular velocity of connecting rod} = \frac{d\beta}{dt}$$

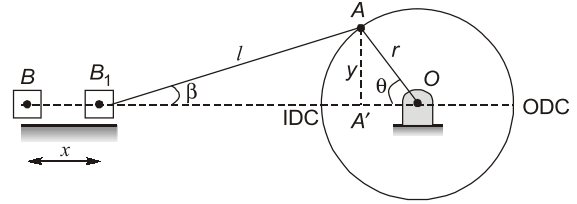
$$\Rightarrow \omega_c = \frac{\omega \cdot \cos\theta}{n \times \frac{\sqrt{n^2 - \sin^2 \theta}}{n}} \quad (\text{from equation (i)})$$

$$\Rightarrow \omega_c = \frac{\omega \cos\theta}{\sqrt{n^2 - \sin^2 \theta}}$$

Let, α_c = angular acceleration of the connecting rod.

$$\begin{aligned} \alpha_c &= \frac{d\omega_c}{dt} = \frac{d\omega_c}{d\theta} \cdot \frac{d\theta}{dt} = \frac{d\omega}{d\theta} (\cos\theta (n^2 - \sin^2 \theta)^{-1/2}) \omega \\ &= \omega^2 (-\cos\theta \cdot \frac{1}{2} (n^2 - \sin^2 \theta)^{-3/2} \cdot (-2 \sin\theta \cos\theta) + (n^2 - \sin^2 \theta)^{-1/2} \cdot (-\sin\theta)) \\ &= \omega^2 \sin\theta \frac{(\cos^2 \theta - (n^2 - \sin^2 \theta))}{(n^2 - \sin^2 \theta)^{3/2}} \\ \alpha_c &= -\omega^2 \sin\theta \left[\frac{n^2 - 1}{(n^2 - \sin^2 \theta)^{3/2}} \right] \end{aligned}$$

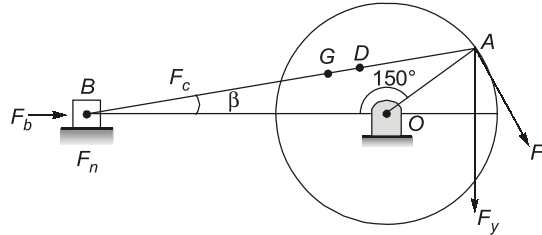
The negative sign indicates that the sense of angular acceleration of the rod is such that it tends to reduce the angle β .



- 3.4** The piston diameter of an internal combustion engine is 100 mm and the stroke is 200 mm. The connecting rod is 4.5 times the crank length and has a mass of 50 kg. The mass of the reciprocating parts is 30 kg. The centre of mass of the connecting rod is 170 mm from the crank pin centre and the radius of gyration about an axis through the centre of mass is 150 mm. The engine runs at 300 rpm. Find the magnitude and the direction of the inertia force and the corresponding torque on the crankshaft when the angle turned by the crank is 150° from the inner dead centre.

(20 Marks)

Solution:



Given: $r = \frac{200}{2} = 100$ mm, $N = 300$, $d = 100$ mm, $l = 450$ mm, $\omega = \frac{2\pi N}{60} = 31.416$ rad/s, $k = 150$ mm

Divide the mass of connecting rod into two parts,

$$\text{Mass at crank pin, } m_a = 50 \left(\frac{450 - 170}{450} \right) = 31.11 \text{ kg}$$

$$\text{Mass at gudgeon pin, } m_b = 50 - 31.11 = 18.89 \text{ kg}$$

$$\text{Total mass of reciprocating parts} = 30 + 18.89 = 48.89 \text{ kg}$$

Accelerating of reciprocating parts, $a = r\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$ as θ is more than 90° , it is negative or towards left and thus inertia force is towards right.

$$\begin{aligned} \text{Inertia force, } F_b &= ma = (48.89)(0.1)(31.416)^2 \left[\cos 150^\circ + \frac{\cos 300^\circ}{4.5} \right] \\ &= (4825.16)(-0.7549) = -3642.58 \text{ N} \end{aligned}$$

Inertia torque due to reciprocating parts,

$$T_b = Fr \left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right] = -146.86 \text{ N.m}$$

(Clockwise as inertia force is towards right)

Correction couple due to assumed second mass of connecting rod at A,

Here,

$$b = 450 - 170 = 280 \text{ mm}$$

$$l = 450 \text{ mm}$$

$$L = b + \frac{k^2}{b} = 360.35 \text{ mm}$$

$$\Delta T = m\alpha_c(b)(l - L)$$

$$\alpha_c = -\omega^2 \sin\theta \left(\frac{n^2 - 1}{(n^2 - \sin^2\theta)^{3/2}} \right) = -104.15 \text{ rad/s}^2$$

$\therefore$

$$\begin{aligned} \Delta T &= (50)(-104.15)(0.280)(0.45 - 0.360) \\ &= -131.23 \text{ N.m} \end{aligned}$$

Direction of the correction couple will be in the direction of decreasing angle β . (Clockwise)

∴ Correction torque on the crankshaft,

$$T_c = \frac{\Delta T \cdot \cos \theta}{\sqrt{n^2 - \sin^2 \theta}} = (-131.23) \frac{\cos 150^\circ}{\sqrt{4.5^2 - \sin^2 150^\circ}} \\ = 25.413 \text{ N.m (CCW)}$$

Torque due to weight of mass at A,

$$T_a = (m_a)r \cos \theta = -26.43 \text{ N.m} \quad (\text{Counter clockwise})$$

$$\therefore \text{Total inertia torque on crankshaft} = T_b(\text{CCW}) + T_c(\text{CCW}) + T_a(\text{CCW}) \\ = -146.86 - 25.413 - 26.43 = 198.703 \text{ N.m}$$

4. Cams and Follower

Level-1

4.1 For a cam whose acceleration is constant and is rotating with a constant speed, plot and show displacement diagram, velocity diagram, acceleration diagram and jerk diagram during rise. (8 Marks)

Solution:

For a constant acceleration cam, the cam accelerate in the first half of rise and decelerate with same value in the second half.

For follower displacement.

$$\text{Displacement} \quad s = \frac{1}{2}ft^2$$

$$\text{Acceleration,} \quad f = \frac{2s}{t^2} = \text{constant}$$

At halfway motion

$$s = \frac{h}{2} \quad \text{and} \quad t = \frac{\phi/2}{\omega} \\ f = \frac{2h/2}{(\phi/2\omega)^2} = \frac{4h\omega^2}{\phi^2}$$

Velocity is linear given by

$$\frac{ds}{dt} = ft = \frac{4h\omega^2}{\phi^2} \frac{\theta}{\omega} = \frac{4h\omega\theta}{\phi^2}$$

