7 *Years*

Previous Solved Papers

GATE 2027

Engineering Sciences

- Fully solved with explanations
- Analysis of previous papers
- Yearwise presentation

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GATE - 2027

Engineering Sciences

Topicwise Previous GATE Solved Papers (2020-2026)

Edition

1st Edition : 2026

Preface

Over the period of time the GATE examination has become more challenging due to increasing number of candidates. Though every candidate has ability to succeed but competitive environment, in-depth knowledge, quality guidance and good source of study is required to achieve high level goals.

B. Singh (Ex. IES)

At the beginning of each subject, analysis of previous papers are given to improve the understanding of subject.

I have true desire to serve student community by way of providing good source of study and quality guidance. I hope this book will be proved an important tool to succeed in GATE examination. Any suggestions from the readers for the improvement of this book are most welcome.

B. Singh (Ex. IES)
Chairman and Managing Director
MADE EASY Group

GATE 2027

Engineering Sciences

CONTENTS

Solid Mechanics

UNIT I

Syllabus

Mechanics of rigid bodies : Equivalent force systems; free-body diagrams; equilibrium equations; analysis of determinate trusses and frames; friction; principle of minimum potential energy; particle kinematics and dynamics; dynamics of rigid bodies under planar motion; law of conservation of energy; law of conservation of momentum.

Mechanics of defformable bodies : Stresses and strains; transformation of stresses and strains, principal stresses and strains; Mohr's circle for plane stress and plane strain; generalized Hooke's Law; elastic constants; thermal stresses; theories of failure.

Axial force, shear force and bending moment diagrams; axial, shear and bending stresses; combined stresses; deflection (for symmetric bending); torsion in circular shafts; thin walled pressure vessels; energy methods (Castigliano's theorems); Euler buckling.

Vibrations: Free vibration of undamped single degree of freedom systems.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2020	9	13	35
2021	9	13	35
2022	9	13	35
2023	9	13	35
2024	9	13	35
2025	9	13	35
2026	9	13	35

1 Solid Mechanics

1.1 Which among the following statements is true for a body moving on a dry surface under the action of applied forces?
(a) Kinetic-friction force is zero.
(b) Kinetic-friction force is equal to the static-friction force.
(c) Kinetic-friction force is greater than the static-friction force.
(d) Kinetic-friction force is lower than the static-friction force.

[2020 : 1 M]

1.2 Consider an isotropic material with Young's modulus E and Poisson's ratio v. The bulk modulus of this material is given by __________.

(a) $\frac{E}{(1-v^2)}$ (b) $\frac{E}{2(1+v)}$

(c) $\frac{E}{3(1-2v)}$ (d) $\frac{Ev}{(1+v)(1-2v)}$

[2020 : 1 M]

1.3 A body subjected to _________ does not undergo change in volume.
(a) uniform tension
(b) pure shear
(c) pure bending
(d) hydrostatic pressure

[2020 : 1 M]

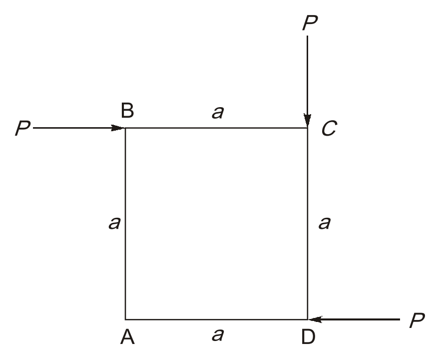
1.4 The angular momentum of a particle moving under a central force is
(a) zero.
(b) constant in both magnitude and direction.
(c) constant in magnitude but not direction.
(d) constant in direction but not magnitude.

[2020 : 1 M]

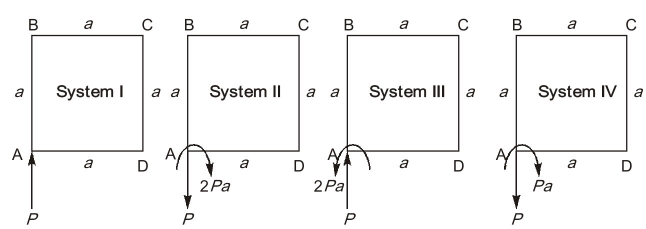
1.5 According to Euler-Bernoulli beam theory, which one of the following statements best describes the state of a beam subjected to pure bending?
(a) Transverse shear stress and transverse shear strain are zero.
(b) Transverse shear stress is not zero but transverse shear strain is zero.
(c) Transverse shear stress is zero but transverse shear strain is not zero.
(d) Transverse shear stress and transverse shear strain are not zero.

[2020 : 1 M]

1.6 A rigid square ABCD is subjected to planar forces at the corners as shown.



For this planar force system, the equivalent force couple system at corner A can be represented as



(a) System I (b) System II
(c) System III (d) System IV

[2020 : 1 M]

1.7 A particle of mass 0.1 kg, which is released from rest, falls vertically downward under gravity in a fluid. The fluid offers a resistive force, which is linearly proportional to the particle velocity with 0.1 N.s/m as the constant of proportionality. The uniform gravitational acceleration is 10 m/s^2 throughout the trajectory of the particle. The magnitude of the particle velocity (in m/s) at time 1 s after release (rounded off to two decimal places) is ______.

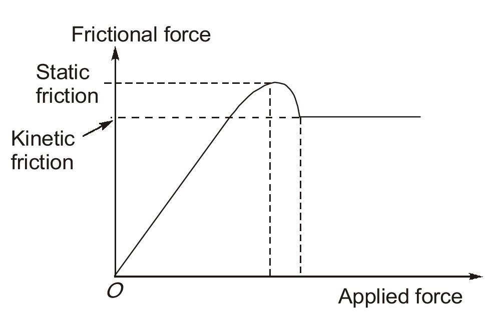
[2020 : 1 M]

Answers Solid Mechanics

1.1 (d)	1.2 (c)	1.3 (b)	1.4 (b)	1.5 (a)	1.6 (b)	1.7 (6.32)
1.8 (90)	1.9 (150)	1.10 (d)	1.11 (a)	1.12 (b)	1.13 (d)	1.14 (a)
1.15 (10)	1.16 (18.75)	1.17 (4.482)	1.18 (7)	1.19 (10.186)	1.20 (5.625)	1.21 (0.182)
1.22 (1.96)	1.23 (d)	1.24 (b)	1.25 (a)	1.26 (b)	1.27 (b)	1.29 (c)
1.30 (a)	1.31 (14.3)	1.32 (c)	1.33 (a)	1.34 (b)	1.35 (a)	1.36 (a)
1.37 (b)	1.38 (30)	1.39 (9)	1.40 (56.57)	1.41 (90.63)	1.42 (4.19)	1.43 (0.87)
1.44 (0.01)	1.45 (a)	1.46 (c)	1.47 (a)	1.48 (b)	1.49 (a)	1.50 (b)
1.51 (c)	1.52 (2)	1.53 (19.23)	1.54 (a)	1.55 (d)	1.56 (a)	1.57 (a)
1.58 (b)	1.59 (a)	1.60 (300)	1.61 (40)	1.62 (1)	1.63 (80)	1.64 (6000)
1.65 (125)	1.66 (150)	1.67 (b)	1.68 (b)	1.69 (a)	1.70 (b)	1.71 (a)
1.72 (a)	1.73 (117.6)	1.74 (5)	1.75 (25)	1.76 (d)	1.77 (c)	1.78 (c)
1.79 (d)	1.80 (a)	1.81 (c)	1.82 (1.9)	1.83 (24)	1.84 (57.3)	1.85 (80)
1.86 (200)	1.87 (2)	1.88 (0.37)	1.89 (d)	1.90 (a)	1.91 (b)	1.92 (0.67)
1.93 (a)	1.94 (d)	1.95 (a)	1.96 (a, b, c)	1.97 (1)	1.98 (25)	1.99 (c)
1.100 (d)	1.101 (c)	1.102 (b)	1.103 (a, c)	1.104 (100)	1.105 (42)	1.106 (0.83)
1.107 (0.31)	1.108 (60)	1.109 (1.2)	1.110 (30)	1.111 (10)	1.112 (d)	1.113 (b)
1.114 (45)	1.115 (a)	1.116 (d)	1.117 (b, c)	1.118 (492)	1.119 (0.4)	1.120 (400)
1.121 (a)	1.122 (c)	1.123 (b)	1.124 (d)	1.125 (c)	1.126 (a, b, d)	1.127 (a, c)
1.128 (605.75)	1.129 (148.21)	1.130 (5)	1.131 (3.45)	1.132 (393)	1.133 (0.005)	1.134 (c)
1.135 (a)	1.136 (a)	1.137 (b)	1.138 (b)	1.139 (a, c)	1.140 (5)	1.141 (86.02)
1.142 (16)	1.143 (d)	1.144 (b)	1.145 (b)	1.146 (a)	1.147 (d)	1.148 (b)
1.149 (b)	1.150 (67.3)	1.151 (15)	1.152 (13)	1.153 (3.25)	1.154 (– 0.027)	

Explanations Solid Mechanics

1.1 (d)



Kinetic friction force < Static friction force

1.2 (c)

Bulk modulus, $K = \dfrac{E}{3(1-2\upsilon)}$

1.3 (b)

A body subjected to pure shear does not undergo any volume change.

1.4 (b)

For a particle moving under a central force, angular momentum remains constant i.e. conserved and torque acting on it is zero. It is because the central force acts along the line connecting the particle and the centre of force. Thus, this central force develops no torque about the centre thereby leading to constant angular momentum.

1.5 (a)

- In Euler - Bernoulli's beam theory, transverse shear strain is zero because it is assumed that plane section remains normal to the neutral axis even after deformation.
- Further, Euler - Bernoulli's beam theory assumes that shear deformations are zero and thus transverse shear stress is zero.

1.6 (b)

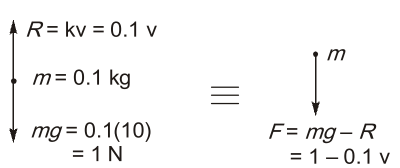
Axial (horizontal) forces at B and D will cancel each other and thus the only unbalanced axial force left is P at corner C which can be transferred to B acting in the downward direction.

Further, axial force acting at D is passing through B and thus it will not develop any couple around B.

∴ Net couple developed at B = Pa + Pa = 2Pa (clockwise)

Thus, answer is (B) i.e. system II.

1.7 Sol.



Net downward force acting on mass, F = 1 – 0.1v

But, $F = ma = 0.1\dfrac{dv}{dt}$

Thus, $F = 1 - 0.1v$

$$\Rightarrow \quad 0.1\frac{dv}{dt} = 1 - 0.1v$$

$$\Rightarrow \quad \frac{dv}{dt} = 10 - v$$

$$\Rightarrow \quad \frac{dv}{(10-v)} = -dt$$

Integrating,

$$\Rightarrow \quad \int_0^v \frac{dv}{(10-v)} = -\int_0^1 dt$$

$$\Rightarrow \quad \ln(10-v)\Big|_0^v = -1$$

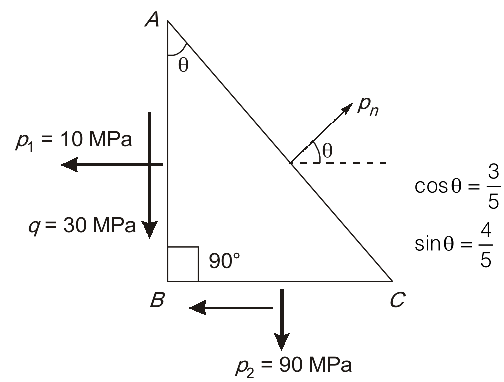
$$\Rightarrow \quad \ln\left(\frac{10-v}{10}\right) = -1$$

$$\Rightarrow \quad \left(\frac{10-v}{10}\right) = e^{-1}$$

$$v = 6.321 \text{ m/s} \simeq 6.32 \text{ m/s}$$

which lies between 6.25 m/s and 6.40 m/s.

1.8 Sol.



$$p_n = \left(\frac{p_1 - p_2}{2}\right) + \left(\frac{p_1 - p_2}{2}\right)\cos 2\theta + q\sin 2\theta$$

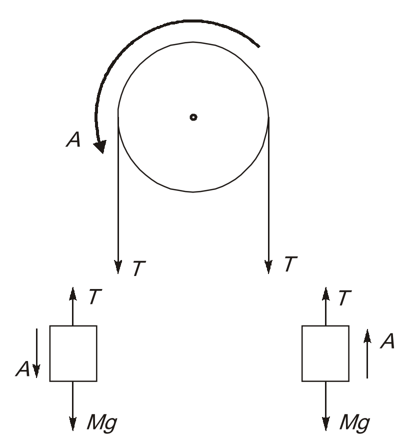
$$= \left(\frac{10+90}{2}\right) + \left(\frac{10-90}{2}\right)(\cos^2\theta - \sin^2\theta) + 30(2\sin\theta\cos\theta)$$

$$= 50 - 40\left(\frac{9}{25} - \frac{16}{25}\right) + 60\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$

$$= 50 + 11.2 + 28.8$$

$$= 90 \text{ MPa}$$

1.9 Sol.



Let A = Acceleration of the system

$Mg - T = ma$ ...(i)

$T - mg = mA$ ...(ii)

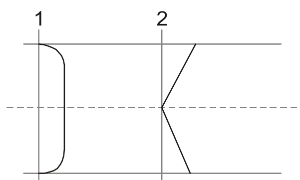
Adding (i) and (ii)

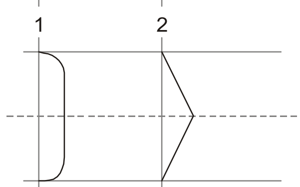
$$(M - m)g = (M + m)A$$

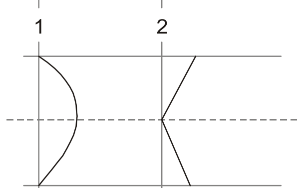
2 Fluid Mechanics

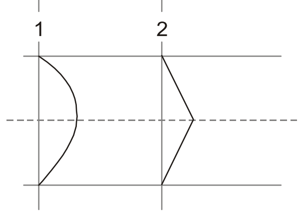
1.1 Figures given below show the velocity and shear stress profiles for the flow in a duel. In each option. T represents velocity profile and '2' represents shear stress profile.

Choose the correct option that closely represents the turbulent tlow condition.

(a)


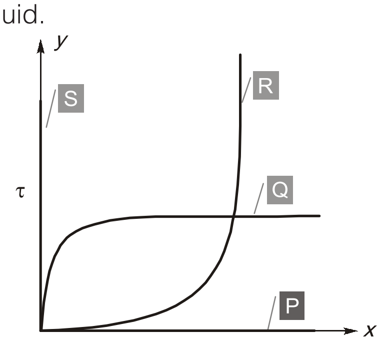
(b)


(c)


(d)


[2020 : 1 M]

1.2 The variation of shear stress (r) against strain rate (du/dy) is given in the Figure. Identify the line/ curve among P, Q, R and S, that represents an ideal fluid.



(a) S
(b) P
(c) Q
(d) R

[2020 : 1 M]

1.3 A body is under stable equilibrium in a homogeneous fluid, where CG and CB are center of gravity and center of buoyancy, respectively. Two statements 'P' and 'Q' are given below:

P: For a fully submerged condition, CG should always be below CB

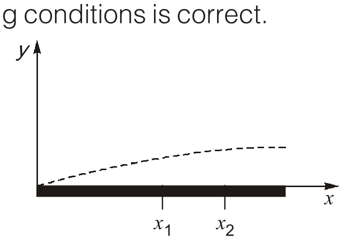
Q: For a floating body, CG need not be below CB.

Choose the option that is valid for the present situation. Which one of the following

(a) P is False; Q is True when metacentre is below CG
(b) P is False; Q is True when metacentre is above CG
(c) P is True; Q is True when metacentre is below CG
(d) P is True; Q is True when metacentre is above CG

[2020 : 1 M]

1.4 A laminar hydrodynamic boundary layer over a smooth flat plate is shown in the Figure. The shear stress at the wall is denoted by t_w. Which one of the following conditions is correct.



(a) Pressure is varying along 'x' and $(\tau_w)_{x1} > (\tau_w)_{x2}$
(b) Pressure is constant along 'x' and $(\tau_w)_{x2} > (\tau_w)_{x1}$
(c) Pressure is constant along 'x' and $(\tau_w)_{x1} > (\tau_w)_{x2}$
(d) Pressure is varying along 'x' and $(\tau_w)_{x2} > (\tau_w)_{x1}$

[2020 : 1 M]

1.5 A non-dimensional number known as Weber number is used to characterize which one of the following flows.

(a) Motion of fluid in open channel
(b) Motion of fluid droplets
(c) Motion of fluid al high velocity
(d) Motion of fluid through a pipe

[2020 : 1 M]

Explanations Fluid Mechanics

1.1 (a)

For a turbulent flow the velocity profile is logarithmic and shear stress profile is linear (maximum at pipe wall and zero at the centre).

1.2 (b)

Ideal fluid show zero resistance against flow.

$\tau = 0$

1.3 (d)

Condition of Stable Equilibrium:

Partially submerged body	Fully submerged body
CG below CB	M should above G

1.4 (c)

For a laminar flow over a flat plate:

$$\tau \propto \frac{1}{\sqrt{x}}$$

$$\tau_w\big|_{x2} < \tau_w\big|_{x1}$$

- Pressure is constant along x.

1.5 (b)

$$\text{Waber number} = \sqrt{\frac{\text{Inertia Force}}{\text{Surface Tension Force}}}$$

- Surface tension is the dominating force. Fluid droplet is the application of the surface tension.

1.6 (a)

Locus of various fluid particles that have passed continuously through a fixed point in a flow field is called streak line. Line R representing the streakline.

1.7 (d)

Discharge between two streamline:

$$Q = |\psi_1 - \psi_2|$$

1.8 Sol.

Given, $d = 0.05$ m; Time = 40 sec; $h = 0.1$ m

$$Ax \text{ velocity} = \frac{\text{Volume}}{\text{Time}}$$

$$\frac{\pi}{4} \times (0.5)^2 \times c = \frac{0.5 \times 0.5 \times 0.1}{40}$$

$$c = 0.31 \text{ m/s}$$

1.9 Sol.

$$\phi = \frac{x^3}{3} - xy^2$$

$$u = -\frac{\partial \phi}{\partial x} = x^2 - y^2$$

at (2, 1)

$$u = 2^2 - 1^2 = 3 \text{ m/s}$$

$$v = -\frac{\partial \phi}{\partial y}$$

$$v = -2xy$$

at (2, 1)

$$v = -4 \text{ m/s}$$

$$|V| = \sqrt{u^2 + v^2} = \sqrt{3^2 + (-4)^2}$$

$$|V| = 5 \text{ m/s}$$

1.10 (c)

Column-I	Column-II
P. Source, Sink, Uniform flow	Rankine oval
Q. Doublet, Uniform flow	Cylinder
R. Source, Uniform flow	Rankine Half body
S. Doublet, Free vortex, Uniform flow	Rotating cylinder

1.11 (b)

Given : $\bar{V} = 5t\hat{i} + 2xz\hat{j} + 2ty\hat{k}$

$u = 5t;\ w = 2ty;\ v = 2xz$

$$\left[a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}\right]$$

$$a_x = 0 + 0 + 0 + 5 = 5$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_y = 5t \times 2z + 0 + 2ty \times 2x + 0$$

$$a_y = (10z + 4xy)t$$

3 Thermodynamics

1.1 If x and y are two independent intensive properties of a thermodynamic system, then which relation among the followings fails to identify z as another thermodynamic property?

(a) $dz = xdy + ydx$ (b) $dz = xdy - ydx$

(c) $dz = 2dy + dx$ (d) $dz = \frac{dy}{x} - \frac{ydx}{x^2}$

[2020 : 1 M]

1.2 Internal energy of a thermodynamic system is defined by the

(a) zeroth law of thermodynamics
(b) first law of thermodynamics
(c) second law of thermodynamics
(d) third law of thermodynamics

[2020 : 1 M]

1.3 In a polytropic process described by PV^n = constant, if $n = 0$, the process is called as

(a) isobaric (b) isochoric
(c) isothermal (d) isentropic

[2020 : 1 M]

1.4 The relation between the coefficient of performance of a refrigerator $(COP)_R$ and the coefficient of performance of a heat pump $(COP)_{HP}$ is

(a) $(COP)_{HP} = (COP)_R + 1$
(b) $(COP)_{HP} = (COP)_R - 1$
(c) $(COP)_{HP} = 1 - (COP)_R$
(d) $(COP)_{HP} \times (COP)_R = 1$

[2020 : 1 M]

1.5 If L_1, L_2 and L_3 are the latent heats of vaporization at the critical temperature of nitrogen, water and ammonia, respectively, then which one of the following is true?

(a) $L_1 > L_2 > L_3$
(b) $L_1 > L_2$ and $L_2 = L_3$
(c) $L_1 < L_2 < L_3$
(d) $L_1 = L_2 = L_3$

[2020 : 1 M]

1.6 A new temperature scale (°N) has been proposed where the normal freezing and normal boiling points of water are marked as 500 °N and 100 °N, respectively. If the temperature of a system is measured to be 0 °N, its temperature according to the Celsius scale (in °C) is ______.

[2020 : 1 M]

1.7 Let Z_1 represents the compressibility factor of air at 2 bar and 600 K, and Z_2 represents the compressibility factor of air at 1 bar and 300 K. If air is assumed to be an ideal gas having gas constant of 0.287 kJ/kg·K, then Z_1/Z_2 is _______.

[2020 : 1 M]

1.8 The rate of heat received by a heat engine from a source at 900 K is 600 kJ/s. The engine rejects heat to the sink of 300 K. The heat engine produces a power of 200 kW. The irreversibility rate (in kW) of the process is _______.

[2020 : 1 M]

1.9 An engine working on the air standard Diesel cycle has a compression ratio of 18. The cycle has a cut-off ratio of 1.7. If the ratio of specific heats of air is 1.4, then the thermal efficiency (in %) of the cycle (rounded off to 1 decimal place) is _____.

[2020 : 1 M]

1.10 A system with rigid walls is initially at a temperature of T_1. It is used as the heat source for a heat engine, which rejects heat to a reservoir maintained at T_0 ($T_0 < T_1$). The specific heats of the system are constant. If the temperature of the system finally reduces to T_0, then the maximum work recoverable from the heat engine per unit mass of the system is

(a) $c_v\left[(T_1 - T_0) - T_0 \ln\left(\frac{T_1}{T_0}\right)\right]$

(b) $c_v(T_1 - T_0)$

(c) $c_v T_0 \ln\left(\frac{T_1}{T_0}\right)$

(d) $c_v \frac{T_1^2}{T_0}$

[2020 : 2 M]

Answers Thermodynamics

1.1	(b)	1.2	(b)	1.3	(a)	1.4	(a)	1.5	(d)	1.6	(125)
1.7	(1)	1.8	(200)	1.9	(64.61)	1.10	(a)	1.11	(d)	1.12	(b)
1.13	(a)	1.14	(a)	1.15	(c)	1.16	(1.2)	1.17	(595.27)	1.18	(2612.5)
1.19	(1.23)	1.20	(39.9)	1.21	(0.54)	1.22	(2.35)	1.23	(c)	1.24	(b)
1.25	(c)	1.26	(d)	1.27	(b)	1.28	(c)	1.29	(c)	1.30	(b, c)
1.31	(160.3)	1.32	(b)	1.33	(a)	1.34	(c)	1.35	(a)	1.36	(106.123)
1.37	(0.3178)	1.38	(13)	1.39	(420)	1.40	(321.53)	1.41	(59.83)	1.42	(91.219)
1.43	(774.9)	1.44	(d)	1.45	(c)	1.46	(a)	1.47	(b)	1.48	(b)
1.49	(753.35)	1.50	(79.45)	1.51	(254.84)	1.52	(25)	1.53	(300)	1.54	(c)
1.55	(a)	1.56	(b)	1.57	(a)	1.58	(b)	1.59	(c)	1.60	(0.22)
1.61	(133.11)	1.62	(4.57)	1.63	(0.0161)	1.64	(0.142)	1.65	(16.06)	1.66	(21.47)
1.67	(d)	1.68	(d)	1.69	(a)	1.70	(b)	1.71	(c)	1.72	(d)
1.73	(20.87)	1.74	(420)	1.75	(4387.535)	1.76	(d)	1.77	(a)	1.78	(b)
1.79	(c)	1.80	(87.34)	1.81	(2976.2)	1.82	(0.927)	1.83	(8.16)	1.84	(2.49)
1.85	(68.449)	1.86	(1.67)	1.87	(2338.71)	1.88	(0.289)	1.89	(c)	1.90	(b)
1.91	(c)	1.92	(a)	1.93	(c)	1.94	(a)	1.95	(b, c)	1.96	(88)
1.97	(0.249)	1.98	(c)	1.99	(d)	1.100	(c)	1.101	(b)	1.102	(d)
1.103	(b)	1.104	(28)	1.105	(0)	1.106	(49.89)	1.107	(271.991)	1.108	(10.806)
1.109	(360)	1.110	(42.717)	1.111	(b)	1.112	(c)	1.113	(a)	1.114	(c)
1.115	(c)	1.116	(a)	1.117	(0.032)	1.118	(80)	1.119	(b)	1.120	(b)
1.121	(a, d)	1.122	(c, d)	1.123	(7.04)	1.124	(64.67)	1.125	(11.749)	1.126	(1530.74)
1.127	(1.33)	1.128	(9.14)	1.129	(8.61)	1.130	(800)	1.131	(8.94)	1.132	(5352.34)
1.133	(c)	1.134	(b)	1.135	(c)	1.136	(d)	1.137	(d)	1.138	(b, c)
1.139	(a, b)	1.140	(3.9)	1.141	(60)	1.142	(c)	1.143	(d)	1.144	(c)
1.145	(c, d)	1.146	(5.99)	1.147	(430.8)	1.148	(5013.7)	1.149	(227.7)	1.150	(6.44)
1.151	(3.549)	1.152	(769.5)	1.153	(459.24)	1.154	(6.66)				

Explanations Thermodynamics

1.1 (b)

$$dz = xdy - ydx$$

Compare it with $dz = Mdx + Ndy$

z is a TD property if it satisfy:

$$\left.\frac{\partial M}{\partial y}\right|_x = \left.\frac{\partial N}{\partial x}\right|_y$$

$$\text{L.H.S. } \frac{\partial}{\partial x}[-y]_x = -1$$

R.H.S. $\frac{\partial}{\partial x}[x]_y = 1$

LHS $\neq$ RHS

1.2 (b)

Internal energy of a TD system is defined by 1st law of TD.

1.3 (a)

Given, $PV^n = C$

if n = 0 $PV^o = C$

$P = C$ (Isobaric process)

1.4 (a)

If a refrigerator and heat pump working between same temperature limit then,

$$COP|_{HP} = 1 + COP|_R$$

1.5 (d)

At critical point latent heat of vapourization is zero so, nitrogen, water and ammonia will have same latent heat of vapourization.

$$l_1 = l_2 = l_3$$

1.6 Sol.

$$t = a + Pb$$

at freezing point:

$$o = a + 500b \quad \text{...(i)}$$

at boiling point:

$$100 = a + 100b \quad \text{...(ii)}$$

Substract eq. (i) from eq. (ii)

$$-100 = 400b$$

$$b = \frac{-1}{4}, \quad a = 125$$

Now, $t = a + Pb$

$$t = 125 - \frac{1}{4} \times 0$$

$$t = 125°$$

1.7 Sol.

Given, $P_1 = 2$ bar, $P_2 = 1$ bar,

$R = 0.287$ kJ/kg-K, $T_1 = 600$ K, $T_2 = 300$ K

Compressibility factor (z) $= \frac{PV}{n\bar{R}T}$

$$\frac{z_1}{z_2} = \frac{P_1}{T_1} \times \frac{T_2}{P_2}$$

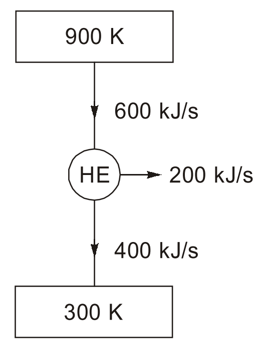
$$\frac{z_1}{z_2} = \frac{2}{600} \times \frac{300}{1}$$

$$\frac{z_1}{z_2} = 1$$

1.8 Sol.

$$I = W_{rev} - W_{act}$$

$$W_{rev} = \left(1 - \frac{T_L}{T_h}\right) Q_{in}$$



$$W_{rev} = \left(1 - \frac{300}{900}\right) \times 600 = 400 \text{ kW}$$

$$I = 400 - 200$$

$$I = 200 \text{ kW}$$

1.9 Sol.

Given, $r = 18,\ r_c = 1.7,\ \gamma = 1.4$

$$\eta_d = 1 - \frac{(r_c^r - 1)}{r(\gamma)^{r-1}(r_c - 1)}$$

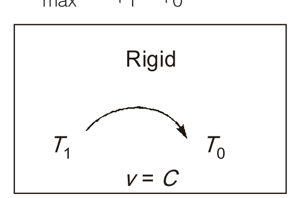
$$\eta_d = 1 - \frac{(1.7)^{1.4} - 1}{1.4(18)^{1.4-1}(1.7 - 1)}$$

$$\eta_d = 0.6461$$

$$\eta_d = 64.61\%$$

1.10 (a)

$$W_{max} = \phi_1 - \phi_0$$



where, $\phi = U - T_0S + P_0V$

ϕ = Availability function

$$W_{max} = (U_1 - T_0S_1 + P_0V_1) - (U_0 - T_0S_0 + P_0V_0)$$

1 Engineering Mathematics

1.1 Let A be a 4 × 3 non-zero matrix and let b be a 4 × 1 column vector. Then Ax = b has
(a) a solution for every b.
(b) no solution for some b.
(c) a solution only when b = 0.
(d) a solution if b and the columns of A form a linearly independent set.
[2020 : 1 M]

1.2 Let A be a 3 × 3 matrix such that $A^2 = A$. Then it is necessary that
(a) A is the identity matrix or the zero matrix
(b) the determinant of A^4 is either 0 or 1
(c) the rank of A is 3
(d) A has one imaginary eigenvalue
[2020 : 2 M]

1.3 The number of points at which the function $f(x, y) = \frac{x^2}{2} + \frac{y^4}{4} - \frac{y^2}{2}$ has local minima is ______.
[2020 : 1 M]

1.4 Let $f(t)$ be a real-valued differentiable function on (–1, 1) such that f(0) = 0 and

$\left|\frac{df}{dt}\right| < 1$ for $0 < t < 1$.

Then the series $\sum_{n=0}^{\infty} f(0.5)^n$
(a) converges but not absolutely
(b) is unbounded
(c) converges absolutely
(d) is bounded but does not converge
[2020 : 2 M]

1.5 Let $\vec{V}(x,y,z) = ax\vec{i} - bz\vec{j} + cy\vec{k}$ be a vector field whose curl is zero. Then necessarily
(a) $a = b = c$ (b) $a = -b = c$
(c) $b = c$ (d) $b = -c$
[2020 : 1 M]

1.6 Let z(t) be the solution of the initial value problem

$$\frac{d^2z}{dt^2} = bz, z(0) = 0, \frac{dz}{dt}(0) = 1 \quad \text{for } t \geq 0.$$

If the planar curve parameterized by t having x - coordinate $z(t)$ and y – coordinate (dz/dt) is closed, then necessarily
(a) b > 0
(b) b < 0
(c) b = 0
(d) b is a non-zero rational number.
[2020 : 1 M]

1.7 Let f(x) be a continuous function on the real line such that for any x,

$\int_0^{x^2} f(t)\,dt = x^2(1 + x^2)$. Then f(2) is ______.
[2020 : 1 M]

1.8 Let z be a complex number. Then the series

$$\sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!}$$

(a) converges for all z.
(b) converges for $|z| \leq 1$ and diverges for $|z| > 1$
(c) converges for z = 0 and diverges for any $z \neq 0$
(d) converges for $|z| < 1$ and diverges for $|z| \geq 1$
[2020 : 1 M]

1.9 Let X be a random variable with probability density function

$$f(t) = \begin{cases} \exp(-t) & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Let 0 < a < b. Then the probability $P(X \leq b, \ | X \geq a)$ depends only on
(a) $b - a$ (b) b
(c) a (d) $a + b$
[2020 : 2 M]

1.10 Players A and B take turns to throw a fair dice with six faces. If A is the first player to throw, then the probability of B being the first one to get a six is____ (round off to two decimal places).
[2020 : 2 M]

Answers Engineering Mathematics

1.1	(b)	1.2	(b)	1.3	(2)	1.4	(c)	1.5	(d)	1.6	(b)	1.7	(5)
1.8	(a)	1.9	(a)	1.10	(0.45)	1.11	(d)	1.12	(d)	1.13	(4)	1.14	(d)
1.15	(c)	1.16	(0.32)	1.17	(0.667)	1.18	(a)	1.19	(b)	1.20	(0.5)	1.21	(0.375)
1.22	(0.5)	1.23	(2)	1.24	(a)	1.25	(c)	1.26	(c)	1.27	(2)	1.28	(3)
1.29	(36)	1.30	(b)	1.31	(4)	1.32	(c)	1.33	(0.067)	1.34	(a)	1.35	(c)
1.36	(b)	1.37	(a)	1.38	(a)	1.39	(3)	1.40	(2)	1.41	(a)	1.42	(0.29)
1.43	(2.63)	1.44	(12)	1.45	(b, c, d)	1.46	(a)	1.47	(3)	1.48	(2)	1.49	(d)
1.50	(2)	1.51	(a)	1.52	(a)	1.53	(b)	1.54	(0.46)	1.55	(c)	1.56	(2)
1.57	(a, c)	1.58	(d)	1.59	(a)	1.60	(d)	1.61	(a)	1.62	(5)	1.63	(0.71)
1.64	(b)	1.65	(d)	1.66	(a)	1.67	(c)	1.68	(d)	1.69	(a)	1.70	(c, d)
1.71	(a, b)	1.72	(256)	1.73	(1)	1.74	(c)	1.75	(a, c)	1.76	(3)	1.77	(1.496)

Explanations Engineering Mathematics

1.1 (b)

A is a 4×3 Non-zero matrix

b is a 4×1 column vector

Linear system, $Ax = b$

$\because$ The system $Ax = b$ has a solution if and only if b lies in the column space of A, which is a subspace of R^4.

Since A is a 4×3 matrix, its column space is a subspace of dimension at most 3 in R^4.

So,

Not all vectors $b \in R^4$ will be in the column space of A.

Hence, there exist vector b for which $Ax = b$ has no solution.

Option (a) $\rightarrow$ False (b may not lie in the column space of A)

Option (b) $\rightarrow$ True (as explained above)

Option (c) $\rightarrow$ False (it may have solutions for some non-zero b)

Option (d) $\rightarrow$ False (b is not part of the matrix).

$\therefore$ The correct answer is option (b)

1.2 (b)

A is a 3×3 matrix such that $A^2 = A$

$\therefore$ It is an idempotent matrix

So, $\det(A^2) = \det(A)$

$(\det(A))^2 = \det(A)$

Let, $\det(A) = x$

$\therefore$ $x^2 = x \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0$

$\therefore$ $x = 0$ or $x = 1$

So, the determinant of matrix A must be either 0 or 1.

$\therefore$ option (b) is correct.

Option (a) is incorrect because there are idempotent matrices that are not identity matrix or the zero matrix.

Option (c) is incorrect because rank of an idempotent matrix can be less than 3.

Option (d) is incorrect because the eigen values of an idempotent matrix are either 0 to 1.

1.3 Sol.

Given:

$$f(x, y) = \frac{x^2}{2} + \frac{y^4}{4} - \frac{y^2}{2}$$

$\therefore$ $f_x = x$

$f_y = y^3 - y$

For critical points,

$f_x = x = 0$

$f_y = y^3 - y = 0 \Rightarrow y(y^2 - 1) = 0$

$\therefore$ $y = 0$, $y = 1$ and $y = -1$

So, the critical points are (0, 0) (0, 1) and (0, –1)

Now,

$$f_{xx} = \frac{\partial}{\partial x}(x) = 1$$

and $$f_{yy} = \frac{\partial}{\partial y}(y^3 - y) = 3y^2 - 1$$

$$f_{xy} = \frac{\partial}{\partial y}(x) = 0$$

$\because$ $$D(x, y) = f_{xx} f_{yy} - (f_{xy})^2 = (1)(3y^2 - 1) - (0)^2 = 3y^2 - 1$$

At (0, 0):

$$D(0, 0) = 3(0)^2 - 1 = -1 < 0$$

$\Rightarrow$ (0, 0) is a saddle point.

At (0, 1):

$$D(0, 1) = 3(1)^2 - 1 = 2 > 0$$

and $f_{xx}(0, 1) = 1 > 0$

$\therefore$ point (0, 1) is a local minimum.

At (0, –1):

$$D(0, -1) = 3(-1)^2 - 1 = 2 > 0$$

and $f_{xx}(0, -1) = 1 > 0$

$\therefore$ point (0, –1) is a local minimum.

$\therefore$ The number of points at which the function has local minima is 2.

1.4 (c)

As $f(t)$ is a differentiable function on (–1, 1), we can apply mean value theorem on the interval [0, 0.5].

There exists a number c in (0, 0.5) such that

$$f'(c) = \frac{f(0.5) - f(0)}{0.5 - 0}$$

$$\Rightarrow \quad f'(c) = \frac{f(0.5)}{0.5} \quad (\text{as } f(0) = 0)$$

$$\Rightarrow \quad f(0.5) = 0.5 f'(c)$$

Also, given that $\left|\frac{df}{dt}\right| < 1$ for $0 < t < 1$.

It implies that $|f'(t)| < 1$ for $t \in (0, 1)$

Since, $c \in (0, 0.5)$, $c \in (0, 1)$ $\Rightarrow$ $|f'(c)| < 1$

$$\therefore \quad |f(0.5)| = |0.5 f'(c)| = 0.5|f'(c)|$$

Since, $|f'(c) < 1|$ $\Rightarrow$ $|f(0.5)| < 0.5(1) = 0.5$

So, $|f(0.5)| < 0.5$

Now, the series $\sum_{n=0}^{\infty} f(0.5)^n$ is geometric with common ratio,

$$r = f(0.5)$$

We got, $|f(0.5)| < 0.5$

$\Rightarrow$ $|f(0.5)| < 1$ $(\because |r| < 1)$

Therefore, the series converges absolutely.

$\therefore$ Option (c) is the correct answer.

1.5 (d)

Given:

$$\vec{V}(x,y,z) = ax\vec{i} - bz\vec{j} + cy\vec{k}$$

$$\text{curl} = 0$$

$\therefore$ $V_x = ax$, $V_y = -bz$, $V_z = cy$

$\because$ **Curl:**

$$\nabla \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

$$\Rightarrow \quad \nabla \times \vec{V} = \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\vec{i} - \left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right)\vec{j} + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\vec{k}$$

$$= (c - (-b))\vec{i} - (0 - 0)\vec{j} + (0 - 0)\vec{k}$$

$$= (c + b)\vec{i}$$

$$\therefore \quad (c + b)\vec{i} = 0$$

$$\Rightarrow \quad c + b = 0$$

$$\Rightarrow \quad b = -c$$

$\therefore$ Correct option is (d)

1.6 (b)

Given:

$$\frac{d^2z}{dt^2} = bz, \; z(0) = 0, = \frac{dz}{dt}(0) = 1 \text{ for } t \geq 0$$

$$\therefore \quad \frac{d^2z}{dt^2} - bz = 0$$

$\therefore$ characteristics equation is $r^2 - b = 0$

$$\Rightarrow \quad r = \pm\sqrt{b}$$

Case 1: $b > 0$

Let $b = k^2$ for some real number $k > 0$. The roots are $r = \pm k$.

General solution is

$$z(t) = c_1 e^{kt} + c_2 e^{-kt}$$

$\because$ $z(0) = 0$ and $z'(0) = 1$,

$\therefore$ $z(0) = c_1 + c_2 = 0 \Rightarrow c_2 = -c_1$